Analysis of Stability in Holographic Resonators
(Análisis de estabilidad en Resonadores Holográficos)

By

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Dedication

This thesis is dedicated to my lovely wife Fedra and my sons: Juan José, Fernanda, Omar y Andrea with all the love of my heart.
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Introduction

1.1 Holographic Laser Oscillator

The beam quality of solid-state laser systems, especially when operating at high-average power, is limited by wavefront distortions caused by thermally-induced aberrations in the active laser materials. A variety of investigations of aberration compensation using nonlinear optical phase conjugation have been conducted, especially using Brillouin scattering. Phase conjugation using inverted gain media has been also well studied to amplify and correct for aberrations inherent in high powered laser beams. The technique used by the authors relies on self-pumped phase conjugation based on an inverted medium in a self-intersecting loop geometry where the gain medium is itself the nonlinear phase conjugator.

![Diagram of Holographic Laser Oscillator](image)

Figure 1.1 Schematic representation of a Holographic Laser Oscillator. The system is constituted by an external laser, an optical isolator, a nonlinear medium (to produce a nonlinear grating), a non-reciprocal transmission element (NRTE), and an amplifier module.
The cavity is formed by a self-intersecting loop and it is constituted by an externally-injected laser signal, which passes through a nonlinear medium where an optically-induced hologram is formed after the signal has completed the loop. The formation of the hologram (or diffractive element) by the interference of the self-intersecting injected beam follows the analysis of optically-induced grating formation that we presented in chapter 4. An optical isolator has to be included in the injecting arm of the system to prevent damage of the external laser. A rather important element in the cavity is the non-reciprocal transmission element (NRTE). This element in the cavity allows for the intensity attenuation of the forward propagating injected beam, which becomes of significant relevance whenever equal beam writing beam intensity is needed to ensure optimum formation of the hologram in the nonlinear medium, and to prevent saturation of the amplifiers (when saturable gain amplifiers are used). Apart from that, the NRTE also ensures unidirectional oscillation of the cavity in the backward direction, which would be the phase conjugate direction if the right phase matching condition are met by the interacting waves. The NRTE is constituted of a pair of Glan-air cube polarisers P₁ and P₂ set to transmit light of horizontal polarisation state; a Faraday rotator is placed between the polarisers together with a half-wave \( \left( \frac{\lambda}{2} \right) \) plate. Right after the NRTE there is the amplifier module, which is the laser active medium in our holographic laser oscillator. The principle of operation of the HLO is as follows: the injected laser signal forms a diffraction grating when it self-intersects in the nonlinear medium. Once the hologram is formed a backward travelling signal is generated from either diffraction of the injected signal or diffraction of the amplified spontaneous emission from the amplifier module. In general, after a few
round trips the backward signal grows sufficient intracavity energy to produce a powerful output.

If the four-wave mixing scheme established between all the interacting beams in the nonlinear medium (the injected, the injected after passing the NRTE and the amplifier, the diffracted, and the diffracted after passing the amplifier and the (NRTE) meet the required matching conditions for phase conjugation, then the backward oscillation of the cavity will have the ability of compensate for intracavity phase distortions and, eventually, for the compensation of polarisation distortions. Successful operation of the HLO depends on a right choice of the nonlinear holographic element. The choice of the nonlinear medium has to be done having into account several considerations. For example, if the amplifier module is a saturable gain medium (Nd:YAG, for instance) and the nonlinear medium is chosen to be a saturable gain/absorption medium, then we have to make sure that the saturation energies or intensities of the two media are comparable. Otherwise it could happen, for example, that the fast saturation of one with respect to the other medium does not allow to achieve a good diffraction efficiency of the holographic element, and hence, to stop the initialisation of the cavity oscillation. Another important consideration that has to be taken into account concerns to the time decay of the nonlinearity of the medium, since a fast time decay of the nonlinearity (Kerr nonlinearity) could result, also, in the stop of the oscillation of the cavity. This time decay of the nonlinearity has to be chosen according to the round-trip time in the loop, specially, when the external signal is from a pulsed source. A third consideration is the possible phase added or subtracted to the diffracted component in the diffraction process due to the nature of the physical mechanism of the nonlinearity. A gain grating gives a $\pi$-phase added factor
to the diffracted component since the gain modulation is $\pi$-off of phase with respect to optical the interference pattern, whereas there is not added phase factor for a diffracted component generated from an incident beam onto an absorption grating which is in phase with the optical interference pattern. A $\frac{\pi}{2}$ phase factor is added to those diffracted components that result from the diffraction of a beam incident into a refractive index grating (a phase grating). This added phase factor to the diffracted beam has to be compensated to ensure resonant oscillation of the cavity. It is also necessary to point out here that some nonlinear media, as Kerr liquids (CS$_2$), can affect significantly the spatial mode operation of the HLO. This is because the intensity-dependent refractive index of that media allows for the spatial structure modification of the beam as a propagation effect. Successful HLO operation has been previously demonstrated for nonlinear gain holographic elements [1,2,3,4]

1.2 Overview of the Thesis

In the chapter 2 started at with the Spatial mode size analysis of a diode-side-pumped Nd:YVO$_4$ laser resonator. when a laser system is to be constructed, it is important to optimize the spatial characteristics of the laser output beam. These spatial mode spot size analysis have been done for self-adaptive holographic laser oscillators in order to determine the self-consistent configuration, as well as the transient evolution and a good control of the mode spot size. Here, we report on a spatial mode spot size analysis of a diode-side-pumped laser resonator that may be thought to operate in continuous wave. The resonator is conformed by a slab of Nd:YVO$_4$ crystal acting as the active laser medium.
The chapter 3 is about the volume grating to operate a holographic laser oscillator. The aim of this chapter is to have the background about the study of several nonlinear media to create a suitable volume grating to operate a self-adaptive laser system.

In the chapter 4, a self-adaptive lasers is constructed by using the four-wave mixing OPC technique in order to induce the formation of a gain grating in an active medium. The gain grating is produced by the gain saturation in the amplifier laser medium obtained through the interference of two coherent intersecting beams. In this way, the self-adaptive laser is created via the diffraction produced by the gain grating when the amplifier medium is located in a self-intersecting geometry (Fig 4.1)

In the chapter 5, an experimental Study of Second-Harmonic Generation by a Laser Pulse with Varying Direction of Polarization in a Type-II Synchronism Doubling Crystal is presented. Finally in the chapter 6 the conclusions are presented
References


Chapter 2:

Spatial mode size analysis of a diode-side-pumped Nd:YVO$_4$ laser resonator.

2.1 Introduction

Recently, diode-pumped Nd-doped crystals have been intensively studied because of their high-power laser emission in the near infrared, 1064 nm, which is extensively used in telecommunications since it coincides with the transmission window of silica optical fibers.

Some of the studied Nd-doped crystals include Nd:YAG [1], Nd:YLF [2], Nd:LSB [3], Nd:GVO [4] and Nd:YVO$_4$ [5]. Mostly, these studies have shown the performance of the above-mentioned Nd-doped crystals in laser action such as light-light conversion efficiency, dopant concentration influence on slope efficiency, maximum output power, Auger upconversion and excited-state absorption-induced losses, etc.

In particular, neodymium-doped yttrium orthovanadate (Nd:YVO$_4$) has been used to construct pulsed lasers [6], end-diode-pumped lasers [7] and optical phase conjugation laser resonators [8]. However, the Nd:YVO$_4$ has poorer thermal properties compared to other Nd-doped crystals like the Nd:YAG, and this leads to thermal lens formation when high-power continuous-wave Nd:YVO$_4$ laser systems are intended to be constructed. Some studies on this material have been presented showing the thermal loading effect on laser action [9].
Apart from the characteristics mentioned before, when a laser system is to be constructed, it is important to optimize the spatial characteristics of the laser output beam. These spatial mode spot size analysis have been done for self-adaptive holographic laser oscillators in order to determine the self-consistent configuration [10], as well as the transient evolution [11] and a good control of the mode spot size [12]. Here, we report on a spatial mode spot size analysis of a diode-side-pumped laser resonator that may be thought to operate in continuous wave. The resonator is conformed by a slab of Nd:YVO₄ crystal acting as the active laser medium, two cylindrical (L₁, L₂) and one spherical (L₃), lenses intracavity for oscillation beam shaping and focusing purposes, a plane mirror (M), and a plane output coupler (OC) (Fig. 2.1).

Fig. 2.1. Schematic of the diode-side-pumped Nd:YVO₄ laser resonator.
2.2 Transfer matrix method for Gaussian beams

Any paraxial ray propagating along a one-dimensional optical system (rotationally symmetric) is fully described at any point of its propagation by its distance from the optical axis of the system, \( r \), and the angle formed between its propagation direction and the optical axis of the system, \( \theta \) (Fig.2.2). These two parameters can be calculated for any position of the propagating ray in the optical system when the transfer matrix of the traversed part of the system, as well as the \( r \) and \( h \) parameters for the ray at the previous position are known, by using the following matrix formalism [13]:

\[
\begin{bmatrix}
    r' \\
    \theta'
\end{bmatrix} = \begin{pmatrix}
    A & B \\
    C & D
\end{pmatrix} \begin{bmatrix}
    r \\
    \theta
\end{bmatrix}
\]

(2.1)

where \( r_0 \) and \( \theta_0 \) are the new parameters of the paraxial propagated ray, and \( A, B, C \) and \( D \) are the elements of the corresponding \( 2 \times 2 \) transfer matrix for the traversed part of the optical system, that is the result of the product of all the transfer matrices representing each of the individual optical elements conforming the part of the optical system traversed by the ray, multiplied in reverse order as it encounters them.

In the case of a paraxial ray propagating through a two-dimensional optical system, non-rotationally symmetric, a good approximation can be obtained by considering the system as having different propagating characteristics for the sagittal and tangential planes. Then, two different distances between the optical axis of the system and the
projections of the paraxial ray propagation direction on the sagittal, $x$, and tangential, $y$, planes have to be considered. In the same way, two different angles are formed between the optical axis of the system and the projections of the ray propagation direction for the sagittal, $\alpha$, and tangential, $\beta$, planes (Fig. 2.3). Therefore, in the case of non-rotationally symmetric optical systems, the preceding matrix formalism described for the one-dimensional optical systems can be generalized by considering a matrix treatment involving $4 \times 4$ transfer matrices as follows [14]:

\[
\begin{pmatrix}
  x' \\
  y' \\
  \alpha' \\
  \beta'
\end{pmatrix} =
\begin{pmatrix}
  A_{ss} & A_{st} & B_{ss} & B_{st} \\
  A_{ts} & A_{tt} & B_{ts} & B_{tt} \\
  C_{ss} & C_{st} & D_{ss} & D_{st} \\
  C_{ts} & C_{tt} & D_{ts} & D_{tt}
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  \alpha \\
  \beta
\end{pmatrix}
\] (2.2)

Where $x'$, $y'$, $\alpha'$ and $\beta'$ are the propagated ray parameters. Some examples of the two-dimensional transfer matrices described above are given in Table 1.

![Paraxial ray propagation](image)

Fig. 2.2. One-dimensional paraxial ray propagation.
This matrix method can also be applied to Gaussian beam propagation by using the complex curvature parameter, \( \bar{q} \), defined by [15]:

\[
\frac{1}{\bar{q}(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi w^2(z)} \tag{2.3}
\]

where \( R(z) \) is the wavefront radius of curvature and \( w(z) \) is the spot size of the Gaussian beam; together with the ABCD Rule [15] written for the sagittal and the tangential directions in the form:

\[
\bar{q}^{'}_{s} = \frac{A_{ss,tt} \bar{q}_{s,t} + B_{ss,tt}}{C_{ss,tt} \bar{q}_{s,t} + D_{ss,tt}} \tag{2.4}
\]

where \( \bar{q}^{'}_{s,t} \) indicates the propagated complex curvature parameter in the sagittal plane when the subscripts are \( s \), or in the tangential plane when the subscripts are \( t \).
Table 1: Examples of two-dimensional transfer matrices

<table>
<thead>
<tr>
<th>Optical Element</th>
<th>Transfer Matrix</th>
</tr>
</thead>
</table>
| Free space propagation over a distance $L$, inside a medium with refractive index $n$ | \[
\begin{pmatrix}
1 & 0 & L/n & 0 \\
0 & 1 & 0 & L/n \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] |
| Tilt on the sagittal plane, with $\theta_i$ and $\theta_t$ the incidence and transmission angles, respectively | \[
\begin{pmatrix}
1 & 0 & (L/n)(\cos \theta_i / \cos \theta_t)^2 & 0 \\
0 & 1 & 0 & L/n \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] |
| Spherical lens with focal length $f$ | \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-1/f & 0 & 1 & 0 \\
0 & -1/f & 0 & 1 \\
\end{pmatrix}
\] |
| Cylinder lens with focal length $f$ in the tangential plane | \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1/f & 0 & 1 \\
\end{pmatrix}
\] |
| Mirror with radius of curvature $R$. | \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-2/R & 0 & 1 & 0 \\
0 & -2/R & 0 & 1 \\
\end{pmatrix}
\] |
2.3 Stability analysis for the diode-side-pumped Nd:YVO$_4$ laser resonator

Any laser resonator can be classified as stable or unstable [16,17]. A laser resonator is said to be stable if for any ray launched inside the resonator parallel to the optical axis, the ray remains inside the resonator after an infinite number of round trips. In terms of transfer matrices parameters, a two-dimensional laser resonator is considered as stable if it satisfies the conditions [18]:

\[-1 < G_{s,t} < 1\]  \hspace{1cm} (2.5)

for both, the sagittal ($G_s$) and the tangential ($G_t$) planes, and with:

\[G_{s,t} = \frac{D_{ss,tt} + A_{ss,tt}}{2}\]  \hspace{1cm} (2.6)

where $A_{ss}$, $A_{tt}$, $D_{ss}$ and $D_{tt}$ are the diagonal elements of the $4 \times 4$ total transfer matrix representing a complete round trip of the Gaussian beam inside the resonator.

The schematic of the studied laser resonator is shown in Fig. 2.1; this was formed by considering a diode-side-pumped Nd:YVO$_4$ crystal slab of $20 \times 5 \times 1$ mm$^3$. The $5 \times 1$ mm$^2$ faces were supposed to be cut at $\approx 2.16^\circ$, and the pumping beam was considered to be focused into the Nd:YVO$_4$ crystal according to the known bounce geometry [19,20]. The focal lengths for the involved lenses in Fig. 2.1 were fixed as 5, 2.54 and 2 cm for L1, L2 and L3, respectively, and the $\phi$ angle was $\approx 4.3^\circ$. The three lenses are
needed in order to reshape the beam to appropriately overlap the pumped region of the amplifier medium, the Nd:YVO\textsubscript{4} crystal in our case.

![Diagram showing paraxial ray propagation]

Fig. 2.3. Two-dimensional paraxial ray propagation.

Since the optical axis of the resonator forms the angle $\phi \approx 4.3^\circ$ with the longitudinal axis of the Nd:YVO\textsubscript{4} crystal in the sagittal plane, a tilt correction in that plane has to be done when calculating the Gaussian beam propagation through the crystal. This correction is done by using the known tilt transfer matrix presented in Table 1 [14,18]. Also, an effective optical length should be taken into account when propagating the Gaussian beam along the path of the resonator inside the crystal; this optical length has to be calculated after considering the refraction of the beam in the surface of the crystal,
with a refractive index of 2 for the Nd:YVO₄ material, and for our case, it resulted to be of ≈20.014 mm.

A transfer matrix analysis was done in order to determine which of the several possible cavity configurations that could be constructed with the fixed set of the lenses and reflectors detailed above would produce stable resonators. For this transfer matrix analysis, the stability parameters of Eq. (2.6) were set to be -0.1 < Gₛ < 0.03 and -0.1 < Gₜ < 0.03, and the five obtained sets of positions are shown in Table 2, each of those representing a different stable cavity configuration for the diode-side-pumped laser resonator.

### 2.4 Mode spot size behavior for the diode-side-pumped Nd:YVO₄ laser resonator

In this section, we report the spatial mode spot size behavior in both the sagittal and tangential planes along the whole trip of the beam inside the resonator. By using the results of Section 2.3, listed in Table 2, each of the cavity configurations were tested by evaluating the spatial mode spot size evolution for a whole trip inside the cavity. Fig. 2.4 shows the mode spot size behavior for the five different configurations of the laser resonator that satisfied the ranges for the stability conditions stated above. In Fig. 2.4, it is possible to observe the values that the mode spot size has at the output coupler position in the resonator, which corresponds to the left-hand side of the plots, for the tangential and sagittal planes. Therefore, it could be determined whether a given
configuration of the resonator is approaching to produce a circular TEM\(_{00}\) mode or not
by analyzing the difference between the mode spot sizes for both planes. It is also
evident in Fig. 2.4 that a suitable choice of the configuration for the resonator may
produce a large spot size together with a relatively good roundness.

Configuration1, listed in Table 2, developed a big difference between the mode spot
size of the output beam for the tangential and the sagittal planes, Fig. 2.4a. A similar
astigmatism for the output beam could be observed for configuration 2, although in this
case, the spot size dimensions in the tangential and sagittal planes are about half the
corresponding spot size dimensions obtained in configuration1 (Fig. 2.4b). However, in
this configuration, the Gaussian beam showed a bigger spot size for the sagittal plane
than for the tangential one while it traverses the Nd\(:\) YVO\(_4\) amplifier slab, which is a

Table 2 The parameters for the five stable resonator configurations (units for distances and resonator's
length are cm)

<table>
<thead>
<tr>
<th>Resonator's Configuration</th>
<th>Stability Parameters</th>
<th>Distances (cm)</th>
<th>Resonator's Length for Single Pass (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sagittal</td>
<td>Tangential</td>
<td>D(_1)</td>
</tr>
<tr>
<td>1</td>
<td>-0.05</td>
<td>0.965</td>
<td>2.3</td>
</tr>
<tr>
<td>2</td>
<td>-0.09</td>
<td>0.034</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-0.08</td>
<td>0.019</td>
<td>2.3</td>
</tr>
<tr>
<td>4</td>
<td>-0.09</td>
<td>0.014</td>
<td>2.1</td>
</tr>
<tr>
<td>5</td>
<td>-0.1</td>
<td>-0.058</td>
<td>2.3</td>
</tr>
</tbody>
</table>
desirable behavior since in such a case, a good overlap of the gain region in the amplifier could be achieved. Configurations 3 and 4 only differ by about 0.1 mm in their mode spot sizes for both planes, as can be observed in Fig. 2.4c and d, respectively. However, a smaller difference between mode spot sizes of the output beam for both, the sagittal and the tangential planes, was obtained for configuration 5; in this case, this difference was of no more than 0.1 mm (Fig. 2.4e).

Fig 2.4a.

Fig. 2.4b
Fig. 2.4c.

Fig. 2.4d.
Fig. 2.4. Spot size behavior of the resonating Gaussian beam, in the tangential and sagittal planes, for the five different cavity configurations satisfying the stated stability conditions, and listed in Table 2 as configuration(a) 1, (b) 2, (c) 3, (d) 4 and (e) 5.

In addition, this configuration also showed to perform a smaller spot size for the tangential plane than for the sagittal one while the beam travels inside the amplifier. Therefore, apart from sensitivity of thermal lens, gain guiding and misalignment, configuration 5 showed to be a good option in order to get a resonator that could produce a good mode spot size of the output beam; the resonator corresponding to configuration 5 is presented in Fig. 2.5.
Fig. 2.5 Stable configuration for the laser resonator that best approximates to produce a circular TEM00 output spatial mode.

2.5 Thermal lens and gain guiding effects in the cavity configuration

In materials with poor thermal properties used in laser resonators or laser amplifiers, like the Nd:YVO₄, it is common to find what is called thermal lens. This problem depends on the local gradient of temperature and is generated by the heating of the active medium produced by the high pumping powers normally used in high-power laser systems.

The net effect of the thermal lens is that of an added lens in the cavity. The focal length of this thermal lens will depend on the deposited heat at a particular point of the gain volume, which then shows that the pumping power will make the original cavity
configuration to change. This change in the original cavity design could probably induce strong astigmatic aberration as well as the degradation of the slope efficiency [9], but the resonator may still be operating well enough for lower pumping powers. Another important cavity design parameter is the overlapping efficiency between the gain volume and the laser mode. Considerable diffraction losses are expected to happen if the laser mode is bigger than the gain volume; on the other hand, if the laser mode is smaller than the gain volume the resonator will be likely to support a multi-mode operation.

In a previous work [21], it has been demonstrated that the absorption depth for a 1.1 at.% Nd:YVO₄ is ≈300 μm, corresponding to an absorption coefficient of α = 30 cm⁻¹. Also in that work, it has been shown that the bouncing geometry can be used to produce high gain in the Nd:YVO₄ amplifier by pumping it with a diode bar focused on the crystal by means of a cylinder lens, whose focal length was 12 mm in that case.

The usual dimensions of the emitting area of such laser devices are 1 x 100 μm², and due to the natural divergence of these diode bar lasers, by freely propagating the beam a few millimeters, it is possible to fill the whole horizontal dimension of the amplifier while having a vertical diameter of ≈200 μm on the plane of the focusing lens after the pumping laser diode beam has been collimated in this fast-diverging axis. Therefore, it is a good approximation to consider that the pumping beam could have a vertical spot size wᵥ = 100 μm on the plane of the cylinder focusing lens, and then estimate that the focused beam spot size inside the amplifier will be in w₀ x 30 μm by using [15]:

\[
W_0 \approx \frac{f\lambda}{\pi w_f}
\]  (7)
where \( f = 12 \text{ mm} \) is the cylinder lens focal length used to focus the pump beam into the Nd:YVO\(_4\) amplifier slab and \( \lambda = 808 \text{ nm} \) is the pumping wavelength. This last calculation will give us the first estimation of the gain volume we should expect inside the amplifier medium, which will then be approximately \( 2 \text{ cm} \approx 60 \mu\text{m} \approx 300 \mu\text{m} \) in length, height and depth, respectively.

As shown in Section 2.4, the mode spot size of the oscillating beam in the crystal for configuration 5 goes from \( \approx 0.1 \text{ to } \approx 0.15 \text{ mm} \) in the tangential plane; however, the gain volume vertical size is expected to be only 60 \( \mu\text{m} \). Therefore, in terms of gain guiding, only the part of the laser mode traveling through the gain volume will be amplified, and a poor overlap between it and the gain volume and low energy extraction efficiency is expected due to the amplifier geometry.

### 2.6 Conclusion

We have done a spatial mode spot size study of a diode-side pumped Nd:YVO\(_4\) laser resonator. The spatial study was done in order to find the best possible stable cavity configuration. The stability analysis of the resonator showed five possible stable configurations for the laser resonator. The mode spot size behaviors for the sagittal and tangential planes along the resonator were obtained for each of the five possible stable configurations found. From the analysis of the mode spot size behavior on both planes, it was possible to determine the best option to build the diode-side-pumped Nd:YVO\(_4\) laser resonator.
2.7 References


Chapter 3

Dynamic gain-gratings

3.1 Introduction

The invention of the laser 35 years ago resulted in powerful light sources which lead into the observation of unexpected phenomena. New fields of science such as holography and nonlinear optics developed rapidly. The classical principle of linear superposition of light waves does not hold anymore when intense beams are used in the interaction with materials that posses significant nonlinear coefficient values.

Conventional laser system are constituted by cavities that use conventional plane or concave mirrors. However, it has been shown[1] that a type of laser system that uses an optically-induced nonlinear diffraction grating (hologram) as one of their mirrors in its cavity has many advantages in the quality of the output, specially when the laser system is operated at high average power. Since the nonlinear optical properties of a holographic element can be inherited to the oscillation of a cavity that uses such an element as a ‘mirror’, it is interesting to dedicate to the study of different nonlinear media in the search of a suitable holographic element to operate a laser system that includes the advantages of nonlinear optics. The aim of this thesis is to perform a systematic experimental study of several nonlinear media to create a suitable volume grating to operate a self-adaptive laser system.

Two laser beams crossing in a suitable material may produce a set of new beams with different direction and frequencies. The optical properties (e.g. refractive index and absorption coefficient) of the matter become spatially modulated (acquiring a grating-
like structure) in the interference region of the two light waves. The spatially modulated optical properties of a particular material can be turned up into permanent diffraction gratings (or long-lived grating structure); or well, in dynamic or transient diffraction gratings [2] where the spatial modulation of the optical properties of the material disappears after irradiation, usually from a laser source, is ended. Laser-induced dynamic diffraction gratings have been created, over the years, in a large number of solids [3,4,5], liquids [6], gases, and atomic vapours [7].

In order to create a laser-induced diffraction grating, two beams are interfered in a suitable material which can modify its optical properties as a result of the interaction. The spatial modulation of the optical properties of the material acts as diffraction grating. The induced changes in the optical properties of a particular material are based in one or more of the different physical mechanisms that are excited by the optical radiation, such as electronic transitions, molecular structural excitations [8] or reorientation [9], or valence-to-conduction band transitions in semiconductors [10], for instance. For most purposes, it is convenient to use collimated TEM$_{00}$ beams, in creating optically-induced gratings, which are closed to an ideal plane wave, and provide plane gratings when brought to intersection.

So far from creating a unique and independent grating in an optical material it comes up that several types of gratings can be created simultaneously. Absorption of light, for instance, populates excited electronic states. Thus, inside the interference pattern, a “population density grating” can be created. During the decay of these electronic states, various lower-energy electronic, vibrational or other states may become populated forming secondary gratings. Finally, the excitation thermalise and thus
inevitably produce a temperature grating. These excitations couple to a refractive index and/or an absorption coefficient [11], i.e., they form optical gratings.

Laser-induced transient diffraction gratings can be used in holography for real time processing of optical fields. This subject has regained interest because of the demonstration of optical phase conjugation or time reversal of optical wave fronts [12]. Using this process, it is possible to design self-adaptive optical systems [1] (described in the introduction or chapter 4) which compensate time-varying phase distortions and polarisation distortions in high-power laser oscillators and amplifiers.

3.2 Laser-induced Diffraction Gratings

As we point out before in the previous section of this chapter, dynamic transient gratings are usually created from the interference of two crossing light beams provided by a laser source in a nonlinear medium. In the formation of the gratings it is common to use a fundamental TEM$_{00}$ mode in order to get a phase front as close as possible to that of a plane wave which is given by:

$$
\bar{E}(\vec{r}, t) = \frac{1}{2} \bar{A} e^{i(\vec{k} \cdot \vec{r} - \omega t)} + c.c. \tag{3.1}
$$

Where $\bar{E}(\vec{r}, t)$ is the electric field, $\bar{A}$ is the field amplitude, $\vec{k}$ is its wave vector, $\omega$ is the frequency of the field, and $\vec{r}$ and $t$ are the usual spatial and time coordinates.

A TEM$_{00}$ mode has a Gaussian rotationally symmetric amplitude distribution,
\[ A(\rho) = A_0 e^{-\frac{\rho^2}{w^2}} \]  

(3.2)

\( \rho \) is the radial coordinate and \( w \) is the spot size of the beam at \( \frac{1}{e} \) of its maximum amplitude. From equation (3.2) we can write the intensity \( I(\rho) \) of a Gaussian beam as follows:

\[ I(\rho) = I_0 e^{-\frac{\rho^2}{w^2}} \]  

(3.3)

In addition to the above equation there are some useful quantities that is convenient to introduce here, they are the total power \( P \) of a TEM\(_{00} \) beam, and the total laser pulse energy. This because short laser pulses are frequently used for grating excitation and detection. The total power is given by:

\[ P = 2\pi \int_0^\infty I(\rho) \rho d\rho = \frac{1}{2} \pi w^2 I_0 \]  

(3.4)

The total laser pulse energy is given by the expression

\[ \varepsilon = 2\pi \int_0^\infty U(\rho) \rho d\rho \]  

(3.5)

where:

\[ U(\rho) = \int_{-\infty}^\infty I dt \]  

(3.6)
is the total pulse energy per unit area, also called the energy fluence.

The experimental arrangement for the production of laser-induced gratings is conceptionally simple although technically it is sometimes quite demanding. The typical set up that is used is shown schematically in figure 3.1. The beam from a laser is split into two beams $I_a$ and $I_b$, with wave vectors $\vec{k}_a$ and $\vec{k}_b$. The two beams intersect at angle $\theta$ at the material sample and create an interference pattern.

![Diagram of laser interference pattern](image)

**Figure 3.1** Optically induced grating formation by interference of two laser beams of intensities $I_a$ and $I_b$, and wave-vector $\vec{k}_a$ and $\vec{k}_b$.

The created grating has a wave-vector given by:

$$\vec{q} = \pm (\vec{k}_a - \vec{k}_b)$$  \hspace{1cm} (3.7)

The spatial period of the grating is linked to the wave-vector by:
\[ \Lambda = \frac{2\pi}{q} \]  

(3.8)

where \( q = 2k \sin \frac{\theta}{2} \) is the absolute value of the grating wave-vector \( \vec{q} \), and \( \theta \) is the crossing angle of the writing beams.

Now, since \( k \) can be written in terms of the wavelength \( \lambda \) of the writing beams as \( k = \frac{2\pi}{\lambda} \), we can rewrite (3.8) as follows

\[ \Lambda = \frac{\lambda}{2 \sin \frac{\theta}{2}} \]  

(3.9)

The electric field amplitude distribution created in the interference region, which leads to an intensity distribution, can be worked out analytically. From equation (3.1) we can write the total electric field in the crossing beams region as follows

\[ \vec{E}(\vec{r}, t) = \vec{E}_a(\vec{r}, t) + \vec{E}_b(\vec{r}, t) = \frac{1}{2} \vec{A}_a e^{i(\vec{k}_a \cdot \vec{r} - \omega t)} + \frac{1}{2} \vec{A}_b e^{i(\vec{k}_b \cdot \vec{r} - \omega t)} \]  

(3.10)

Using the coordinate system shown in figure 3.1 we can write

\[ \vec{k}_a = k_x \vec{x} + k_z \vec{z} \]  

(3.11a)
\[ \vec{k}_b = -k_x \vec{x} + k_z \vec{z} \]  

(3.11b)

furthermore \( \vec{r} = x\vec{x} + z\vec{z} \), so that

\[ \vec{k}_a \cdot \vec{r} = k_x x + k_z z \]  

(3.12a)

\[ \vec{k}_b \cdot \vec{r} = -k_x x + k_z z \]  

(3.12b)

The total electric field at the interference region can be written as

\[ \vec{E}(\vec{r}, t) = \frac{1}{2} \vec{A} e^{i(k_z z - \omega t)} \]  

(3.13)

where

\[ \vec{A} = A_a e^{i k_x x} + A_b e^{i k_x x} \]  

(3.14)

is the electric-field amplitude distribution at the interference region, while the total time-dependent field is given by expression (3.13).

Using (3.14) we can also write an expression for the total intensity distribution. The intensity of an electric field can be written as
\[ I = \frac{1}{2} n \varepsilon_0 c (A \cdot A^*) \] (3.15)

where \( n \) is the linear refractive index, \( \varepsilon_0 \) is the electric permittivity in vacuum and \( c \) is the speed of light also in vacuum, and the star in \( A \) stands for complex conjugate.

Using expression (3.14) in (3.15), we obtain

\[ I = \frac{1}{2} n \varepsilon_0 c \left( |\vec{A}_a|^2 + |\vec{A}_b|^2 + 2 \vec{A}_a \cdot \vec{A}_b^* \cos 2k_x x \right) \]

\[ = I_a + I_b + \frac{1}{2} n \varepsilon_0 c \left( 2 \vec{A}_a \cdot \vec{A}_b^* \cos 2k_x x \right) \] (3.16)

here we can define the quantity

\[ \Delta I = \frac{1}{2} n \varepsilon_0 c \vec{A}_a \cdot \vec{A}_b^* \] (3.17)

which we called from now on the intensity modulation amplitude. So, we can write (3.16) as follows

\[ I = I_a + I_b + 2\Delta I \cos 2k_x x \] (3.18)
We must point out here that the quantity \( \Delta I \) as defined by (3.17) becomes very useful when both the grating medium and the interaction mechanism are isotropic. However, when we are dealing with anisotropic media or an anisotropic interaction, a diffraction grating may be created even if \( \vec{A}_a \) and \( \vec{A}_b \) are perpendicular between them and \( \Delta I = 0 \), which is the case of crossing beams that are orthogonally polarised. In that case, it is convenient to define a new quantity we will call the interference tensor, which will be given by:

\[
\Delta M = \frac{1}{2} n \varepsilon_0 c A_{ai} \cdot A_{bj}^*
\]  

(3.19)

The intensity modulation amplitude \( \Delta I \) is linked to the interference tensor by the absolute value of the trace of \( \Delta M \), i.e.,

\[
\Delta I = |Tr\Delta M|
\]  

(3.20)

There are four cases with different polarisations for which we will give the form of the interference tensor. Consider the superposition of beams with different polarisations as follows:

1) \( \bar{s} \) polarisation: \( \vec{A}_a \parallel \vec{A}_b \parallel \hat{y} \) (see fig 3.3a)

\[
\Delta M = \begin{pmatrix}
0 & 0 & 0 \\
0 & \Delta I & 0 \\
0 & 0 & 0
\end{pmatrix}
\]  

(3.21)
2) \( \bar{\rho} \) polarisation: \( \tilde{A}_a \parallel \tilde{A}_b \perp \hat{y} \)

\[
\Delta M = \frac{1}{2} n \varepsilon_0 c \begin{pmatrix}
A_{ax} A_{bx}^* & 0 & A_{ax} A_{bx}^*
0 & 0 & 0
A_{ax} A_{bx}^* & 0 & A_{ax} A_{bx}^*
\end{pmatrix}
\]  \hspace{1cm} (3.22)

In this case, the intensity modulation is:

\[
\Delta I = |Tr\Delta M| = \frac{1}{2} n \varepsilon_0 c (A_{ax} A_{bx}^* + A_{ax} A_{bx}^*)
\]  \hspace{1cm} (3.23)

It is necessary to note that depending on the relative phase \( \phi_{ab} = qx \) of the two pump beams along \( \hat{x} \)-axis, the superposition of \( \tilde{A}_a \) and \( \tilde{A}_b \) results in a polarisation that varies between linear and elliptic.

The interference field polarisation is particularly interesting if in addition \( |\tilde{A}_a| = |\tilde{A}_b| \). If that is the case, then the polarisation points towards the \( \hat{x} \)-direction for \( \phi_{ab} = 0 \), becomes circular for \( \phi_{ab} = \frac{\pi}{2} \), and finally at \( \phi_{ab} = \pi \), it is linear in the \( \hat{z} \)-direction, i.e., a longitudinal field with regard to the interference pattern.
3) Mixed polarisation: $\vec{A}_a \parallel \hat{y}$ and $\vec{A}_b \perp \hat{y}$.

$$\Delta M = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ A_{ay}A_{bx}^* & 0 & A_{ay}A_{by}^* \\ 0 & 0 & 0 \end{pmatrix}$$  \hspace{1cm} (3.24)

So, $\Delta I = |Tr\Delta M| = 0$ and therefore no intensity modulation exists.

4) Opposite circular polarisations and $|\vec{A}_a| = |\vec{A}_b|$.

$$\Delta M = \frac{1}{2} n\varepsilon_0 c |\vec{A}_a|^2 \begin{pmatrix} \cos^2 \frac{\theta}{2} & \frac{i\cos \theta}{2} & \frac{\sin \theta}{2} \\ \frac{i\cos \theta}{2} & -1 & i\sin \frac{\theta}{2} \\ -\frac{\sin \theta}{2} & -i\sin \frac{\theta}{2} & -\sin^2 \frac{\theta}{2} \end{pmatrix}$$  \hspace{1cm} (3.25)

with intensity modulation

$$\Delta I = |Tr\Delta M| = \frac{1}{2} n\varepsilon_0 c |\vec{A}_a|^2 \sin^2 \theta$$  \hspace{1cm} (3.26)

Until here we have already described the spatial interaction of the two writing beams.

But, when the interfering beams are provided by a pulsed laser of very short duration,
the interference depends necessary on the delay $\tau$ between the pulses. If the time
dependence of the writing pulses is Gaussian of half width $t_p$. Then the magnitude of
the interference tensor, in this case, also depends on the overlap of the two pulses and
is given by

$$
\Delta M_y = \frac{1}{2} n c A_m A_y e^{-\left(\frac{\tau}{2t_p}\right)^2} e^{-\left(\frac{\tau}{t_p}\right)^2}
$$

(3.27)

where the ratio $\frac{\tau}{t_p}$ is the overlapping factor of the two beams.

Thus, the temporal behaviour of $\Delta M(t)$ is the same as that of the original writing
pulses, but its amplitude decreases in proportion to $e^{-\left(\frac{\tau}{2t_p}\right)^2}$.

3.3 Amplitude and Phase Diffraction Gratings

The process of light-induced changes of optical properties of a particular medium can
be understood, in a first step, as the excitations in the medium that are produced by
the interaction light-matter. In the simplest case, the excitations that take place in the
medium lead to the change of ground state absorption coefficient, as well as the
change of the refractive index of the medium that result in amplitude and phase
diffraction gratings.

When a material is placed at the crossing region (interference region) of two laser
beams, some basic processes such as light absorption occurs. As a result of this
absorption process electronic excitations are possible, and hence the creation of a
corresponding population of some excited state is also possible. For instance, if the material is a semiconductor then the absorption may induce a conduction electron density modulation, or in the event of a photorefractive materials then a spatial charge modulation is built-up. When liquids are used as a recording medium for laser-induced diffraction gratings, then temperature or molecular orientation modulation are expected.

Many of these excitations can be described by the population of one, several, or the whole continuum of excited states of the grating medium. Here, the gratings that rise with each one of the excitations can also be considered population gratings in a generalised sense.

The description in terms of the excited-state populations is necessary if the local population density is out of thermal equilibrium. This is usually the case if the excited-state energy is far above the thermal energy \( k_B T \).

Once the absorbed energy is thermalised locally, a description of the resulting grating in terms of the usual thermodynamics variables, temperature, density, etc., becomes appropriate and convenient. The medium as a whole is not in equilibrium as long as these quantities still varying spatially. Their equilibration requires transport of heat, matter, etc., which usually occurs by diffusion. Thus, their decay time depends on the size of the excitation gradients and hence the on the \( \vec{q} \) vector of the grating.

Under stationary conditions, the material excitation amplitude \( \Delta X \) is proportional to the modulated intensity amplitude \( \Delta I \), and in the simplest case

\[
\Delta X = K(\lambda)\Delta I
\]  

(3.28)
where $K(\lambda)$ is a coupling coefficient which depends on the type of material excitation and the pump wavelength $\lambda$.

The material excitation, in general, couples to the refractive index and absorption coefficient which, then, exhibit a grating-like modulation with amplitudes $\Delta n(\lambda)$ and $\Delta \alpha(\lambda)$. Generally speaking, any spatial modulation of a material property with amplitude $\Delta X$ inside a particular medium will be accompanied of an optical grating with amplitudes

$$\Delta n = \left( \frac{dn}{dX} \right) \Delta X$$  \hspace{1cm} (3.29a)

$$\Delta \alpha = \left( \frac{d\alpha}{dX} \right) \Delta X$$  \hspace{1cm} (3.29b)

Instead of using two separate optical parameters, namely the absorption coefficient $\alpha$ and the refractive index $n$, it is often convenient to combine these two quantities to a complex refractive index

$$\tilde{n} = n + i \frac{\alpha}{2k}$$  \hspace{1cm} (3.30a)

$$\Delta \tilde{n} = \Delta n + i \frac{\Delta \alpha}{2k}$$  \hspace{1cm} (3.30b)
where $k$ is the absolute value of the wave-vector of light used in the creation of the grating.

Now, from (3.29a) and (3.29b) we can rewrite

$$
\Delta \vec{n} = \left( \frac{dn}{dX} \right) \Delta X + \frac{i}{2k} \left( \frac{d\alpha}{dX} \right) \Delta X = \left[ \left( \frac{dn}{dX} \right) + \frac{i}{2k} \left( \frac{d\alpha}{dX} \right) \right] \Delta X
$$

$$
= \left[ \frac{d}{dX} \left( n + \frac{i}{2k} \alpha \right) \right] \Delta X = \left( \frac{dn}{dX} \right) \Delta X
$$

which is a general expression for the modulation of the complex refractive index of the medium, i.e., the modulation of the optical properties of a particular medium by light-induced material excitations.

### 3.4 Transient Diffraction Gratings in Nonlinear Optics

The process of grating generation and detection (via a probing beam) involves the interaction of four light beams, in general: two pump beams, the probe beam, and the diffracted beam. Since for nonlinear media the four beams do not linearly superimpose, but they interact in such a way that they influence one to another, the grating formation (and detection) scheme falls into the regime of nonlinear optics.

The general approach to describe nonlinear optical interactions, which has become the classical one, was introduced by Bloembergen [13] and his co-workers in 1962. They started with the local, time-dependent, polarisation $\vec{P}(\vec{r}, t)$, which in general, is a functional of the local and time-dependent electric field. The functional character of the polarisation is of relevance mostly for transient effects; in steady-state and
assuming local response of the medium, the polarisation becomes a function instead a functional of the electric field. The steady-state polarisation can be expanded into a power series of the electric field \( \vec{E} \).

\[
\vec{P}_{nl} = \chi^{(2)} \vec{E} \cdot \vec{E} + \chi^{(3)} \vec{E} \cdot \vec{E} \cdot \vec{E} + \cdots
\]  

(3.31)

When we want to work out the interaction of a certain number of light waves, we have to solve the Maxwell’s equations using the expression for the nonlinear polarisation as a source term. In that case, the nonlinear wave equation takes the form [14]

\[
\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{P}_{nl}
\]  

(3.32)

where \( \vec{E} \) is the total electric field in the wave mixing and \( \vec{P}_{nl} \) is given by (3.31).

The right hand side term in equation (3.32) acts as a source term and only those terms which are phase matched have a non-vanishing contribution to the nonlinear process. Since the grating writing-reading process involves the interaction of four fields, we can identify this process with a third order nonlinear process, i.e., with the second term in expression (3.31). This is because the second order term in the nonlinear polarisation corresponds to a process where the combination of only two fields are involved, and hence, where only the sum or the difference of the two available field frequencies can occur to obtain the generation of a beam of frequency: \( 2\omega_p \).
\( \omega_p \pm \omega_R, 2\omega_R, 0 \). But, there is not any term with frequency \( \omega_d = \omega_R \). Here, \( \omega_p \) is the pump beam frequency, \( \omega_R \) is the reading beam frequency and \( \omega_d \) is the diffracted beam frequency. The same combination is true with the other even-order terms. Thus, we are left to the lowest odd-order, i.e. with the third order term which provides the required combination

\[
\omega_d = \omega_p - \omega_p + \omega_R
\] (3.33)

This combination corresponds to a contribution from the source term, which produce a new field \( \vec{E}_d \) with an amplitude \( \vec{A}_d \), when the phase matching condition

\[
k_d = k_s + k_s + k_d
\] (3.34)

is satisfied.

In that case, the nonlinear polarisation is reduced to:

\[
\vec{P}_d(\vec{r}, t) = \chi^{(3)} \vec{E} \cdot \vec{E} \cdot \vec{E}
\] (3.35)

where, for the sake of simplicity, higher odd-order terms have been neglected. In this manner, all the nonlinear interactions of the during the grating formation and the reading process are, now, expressed in one single quantity, the third order susceptibility \( \chi^{(3)} \). Furthermore, we can work out directly the diffracted field strength
substituting (3.35) into the nonlinear wave equation and writing the total electric field explicitly in terms of the individual components: the pump fields, the probe field, and the diffracted field.

3.5 Population Density Diffraction Gratings

The optical excitation of an atomic system from its ground to an upper-state changes both the ground-state absorption coefficient and the refractive index of the medium that is constituted by such an atomic system. This can be observed in a diffraction grating experiment.

If simplified atomic systems are considered, where only two electronic-energy levels become populated, the change of the absorption coefficient can be worked out from the absorption cross section \( \sigma_0 \) and \( \sigma_1 \) of the ground and the excited states, respectively, which are known experimentally for a certain number of materials. The absorption coefficient \( \alpha \) is given by the population density \( N_0 \) and \( N_1 \) of the two states.

\[
\alpha = \sigma_0 N_0 + \sigma_1 N_1
\]  

(3.36)

By optical excitation, the population densities change to \( N_0 - \Delta N \) and \( N_1 + \Delta N \), giving as a result a change of the absorption coefficient that can be written as:

\[
\Delta \alpha = -(\sigma_0 - \sigma_1)\Delta N
\]  

(3.37)
The change of the absorption coefficient is accompanied by a change of the refractive index $\Delta n$. It is possible to estimate $\Delta n$ from $\Delta \alpha$ by using the Kramers-Kronig formula [15]

$$\Delta n = \frac{1}{2\pi^2} \int_0^\infty \Delta \alpha(\lambda) \left[ 1 - \left( \frac{\lambda'}{\lambda} \right)^2 \right]^{-1} d\lambda'$$ \hspace{1cm} (3.38)

which is valid for small absorption coefficients ($\alpha$ and $\Delta \alpha \ll \frac{\pi}{\lambda}$) and small refractive index changes ($\Delta n \ll n$), $\lambda = 2\pi \frac{c}{\omega}$ is the vacuum wavelength. The change of the absorption coefficient and the refractive index can be combined to express a change of the complex susceptibility. From

$$\chi = \left( n + i \frac{c}{2\omega} \alpha \right)^2 - 1 \approx n^2 - 1 + i \frac{nc}{\omega} \alpha$$ \hspace{1cm} (3.39)

with $\alpha \ll 2 \frac{\omega}{c}$.

From (3.39) we obtain

$$\Delta \chi = 2n \Delta n + i \frac{nc}{\omega} \Delta \alpha$$ \hspace{1cm} (3.40)
The diffraction efficiency $\eta$ of a weak diffraction grating (of thickness $d$) in terms of the susceptibility $\Delta\chi$ becomes

$$\eta = \left( \frac{\pi nd}{\lambda} \right)^2 + \left( \frac{\Delta\alpha d}{4} \right)^2 = \left( \frac{\pi nd}{2\lambda} \right)^2 |\Delta\chi|^2$$

(3.41)

The diffraction efficiency thus measures the change of the absolute value of the complex susceptibility. Because optical excitation changes both the absorption coefficient and the refractive index, a population density diffraction grating is a mixed amplitude and phase grating.
3.6 Four-wave mixing in nonlinear media

Since the optically-induced diffraction grating writing-reading process involves the interaction of four electric fields, as it is shown schematically in figure 3.2, we will present in this section of the chapter a general theory description of the four-wave mixing (FWM) technique in nonlinear optics.

![Diagram](image)

**Figure 3.2** Writing and reading of a transmission diffraction grating by co-polarised beams $A_1$ and $A_3$ (writing beams), and the orthogonally-polarised beam $A_2$ (reading). The diffracted beam $A_4$ has the same polarisation as the reading beam.

The interaction of an incident optical beam with a material medium induces a dipole moment per unit volume called the polarisation. In general, the total polarisation $\vec{P}$ is connected to the total optical electric field $\vec{E}$ by a field-dependent susceptibility $\chi(\vec{E})$ and can be written as follows:

$$\vec{P}(\vec{E}) = \chi(\vec{E})\varepsilon_0\vec{E}$$

(3.42)
The susceptibility of the medium can be expanded in a power series [16] as follows

$$\chi(\vec{E}) = \chi^{(1)} + \chi^{(2)} \vec{E} + \chi^{(3)} \vec{E} \cdot \vec{E} + \cdots \quad (3.43)$$

resulting in the polarisation expression given by

$$\vec{P}(\vec{E}) = \varepsilon_0 (\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E} \cdot \vec{E} + \chi^{(3)} \vec{E} \cdot \vec{E} \cdot \vec{E} + \cdots) \quad (3.44)$$

where the first-order term corresponds to linear optical phenomena such as linear refractive index, gain and absorption. In anisotropic media the quantity $\chi^{(1)}$ becomes a rank two tensor operating on the vector field and describes effects such as birefringence. In general, the $n^{th}$ order susceptibility $\chi^{(n)}$ is a rank $(n+1)$ tensor.

However, for isotropic media the susceptibilities can be treated as scalar quantities.

The $\chi^{(2)}$ term in (3.44) corresponds to second order effects which result in three-wave mixing since a third field is usually produced in addition to the two fields whose product appears in the nonlinear polarisation. Examples of such effects include second harmonic generation $\chi^{(2)}(2\omega;\omega,\omega)$, parametric up (down) conversion $\chi^{(2)}(\omega_1 \pm \omega_2;\omega_1,\pm\omega_2)$, and the Pockels effect $\chi^{(2)}(\omega;\omega,0)$; here the first quantity in brackets is the frequency that results from the combination of the other two.

The third order susceptibility $\chi^{(3)}$ is the term in the nonlinear polarisation that gives rise to four-wave mixing processes which, as will be shown, are well suited to the production of a phase conjugate beam. Third order effects include third harmonic
generation $\chi^{(3)}(3\omega; \omega, \omega, \omega)$, non-degenerate four-wave mixing $\chi^{(3)}(\omega_1 \pm \omega_2 \pm \omega_3; \omega_1, \omega_2, \pm \omega_3)$, stimulated Brillouin and Raman scattering $\chi^{(3)}(\omega \pm \Omega, \omega, -\omega, \omega \pm \Omega)$, and degenerate four-wave mixing $\chi^{(3)}(\omega, \omega, \omega, -\omega)$.

The most common arrangement for achieving four-wave mixing is shown in figure 3.3. It consists of two strong counterpropagating pump beams, $\vec{E}_1$ and $\vec{E}_2$, incident on a $\chi^{(3)}$ nonlinear medium, and a weaker probe beam, $\vec{E}_3$, injected at a small crossing (with beam $\vec{E}_1$) angle $\theta$. The possibility of phase conjugation arises because of the mixing of the three waves given by:

$$E_j = \frac{1}{2} \left[ A_j e^{i(k_j z - \omega_j t)} + c.c. \right]$$  \hspace{1cm} (3.45)

where $j = 1, 2, 3$, can give rise to a nonlinear polarisation with a component

$$P_{nl}(\omega_4 = \omega_1 + \omega_2 - \omega_3) \propto \chi^{(3)} A_1(\omega_1) A_2(\omega_2) A_3^*(\omega_3)$$  \hspace{1cm} (3.47)

which drives a fourth wave, $A_4$, at frequency $\omega_4$.

An alternative physical picture of the wave mixing can be constructed in terms of the laser-induced diffraction gratings. For co-polarised radiation, the probe beam, $\vec{E}_3$, interferes with the two pump beams, $\vec{E}_1$ and $\vec{E}_2$, to produce two separate gratings (transmission and reflection gratings, respectively) within the medium from which the pump beams scatter into the probe ($\vec{E}_3$) and the conjugate ($\vec{E}_4$) directions. The
situation is shown in figure 3.3, indicating the two contributions to the conjugate beam. The probe and the forward pump beam ($\vec{E}_3$) establish a wide period transmission grating from which $\vec{E}_2$ diffracts, whereas the probe and the backward pump beam ($\vec{E}_2$) establish a narrow period reflection grating from which $\vec{E}_1$ reflects. If the two pump beams are cross-polarised then only one of these gratings is formed in the medium, and it depends on the polarisation state of the probe beam with respect to the pump beams polarisation state whether a transmission or a reflection grating is formed. The actual nature of the gratings depends upon the type of nonlinearity giving rise to the $\chi^{(3)}$ susceptibility. This can include the spatial modulation of parameters such as gain [17], absorption [18], refractive index [19], and temperature [20].

![Diagram of transmission and reflection gratings](image)

**Figure 3.3** Transmission and reflection gratings formed between the forward pump and the probe beams, and between the backward pump and the probe beam, respectively.

The whole process is similar to holography\(^{(29)}\) with $\vec{E}_1$ (or $\vec{E}_2$) as the reference beam, $\vec{E}_3$ as the object beam, $\vec{E}_2$ (or $\vec{E}_1$) as the reading beam, and $\vec{E}_4$ as the resulting
image beam. The FWM process is sometimes referred to as real-time or dynamic holo
graphy.

In the particle picture of FWM, the process is viewed as the annihilation of two pump photons ($\omega_1$ and $\omega_2$) to create a probe photon ($\omega_3$) and a signal photon ($\omega_4$). Conservation of energy requires the generated conjugate photon to satisfy the relationship

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

(3.47)

For the special case $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega$, all the interacting waves are at the same frequency and the process is termed degenerate four-wave mixing (DFWM). The efficiency of the interaction is usually optimised if momentum conservation (phase-matching) is also satisfied, i.e.

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4$$

(3.48)

In the wave picture of this requirement equates to Bragg-matching the reading beam to the diffraction grating period. When the counterpropagating pump beam geometry is used in DFWM, such that $\vec{k}_1 = -\vec{k}_2$, the conjugate signal $\vec{E}_4$ retraces the path beam of the probe $\vec{E}_3$ with a wave-vector $\vec{k}_4 = -\vec{k}_3$. As a result, the phase-matching condition is automatically satisfied for all angles of incidence onto the medium by the probe.
3.7 Conventional Phase Conjugation and Vectorial Phase

Conjugation

Optical phase conjugation (OPC) is a remarkable technique by which both the direction and the phase structure of an arbitrary beam of light can be reversed. A device which exhibits such behaviour is known as a phase-conjugate mirror (PCM). The name arises because the ideal action is that of a unique kind of mirror which instantaneously adapts its reflective surface to the wavefront shape of the incident lightwave. This behaviour is entirely equivalent to the mathematical operation of taking the complex conjugate of the spatial phase variation of the lightwave, hence the term phase conjugation. The mathematically description of an optical beam is generally of the form

\[
\bar{E}(\vec{r}, t) = \frac{1}{2} \left[ |A_i(\vec{r})| e^{i(\omega t - k z + \phi(\vec{r}))} + \text{c.c.} \right]
\]  

(3.49)

for a wave travelling in the +z direction with wave-vector \( k \), and frequency \( \omega \) and complex amplitude \( A_i(\vec{r}) = |A_i(\vec{r})| e^{-i\phi(\vec{r})} \), where \( \phi(\vec{r}) \) is the phase of the wavefront.

The phase conjugate beam is then defined as

\[
\bar{E}(\vec{r}, t) = \frac{1}{2} \left[ |A_i(\vec{r})| e^{i(\omega t + k z - \phi(\vec{r}) + \phi(\vec{r}))} + \text{c.c.} \right]
\]  

(3.50)
The properties of a PCM, as described above, immediately suggest its application to the problem of correcting unavoidable aberrations in an optical system. The phase aberration compensation is achieved by the distorted wavefront experiencing a second pass of the aberrating medium after reflection at a PCM. The situation is shown in figure 3.4. This figure shows a plane wave beam incident onto a distorfer \( n(\vec{r}) \) where its phase is distorted; the figure also shows that on reflection at a PCM the aberrated wave is reversed towards the medium, retracing its original path. Consequently, after a second passage of the medium the beam recovers its original phase structure. The source of distortion \( n(\vec{r}) \) could be optical elements, atmospheric turbulence, fibre-optics or a laser amplifier with thermally-induced lensing. For some of these examples the aberration will not be static, that is \( n(\vec{r}) \) would change to \( n(\vec{r},t) \). In that case, it is still possible to compensate for the dynamic aberrations providing that they change on a timescale that is long compared to the sum of the double round trip time and the response time of a real PCM.

![Diagram](image)

**Figure 3.4** Picture of phase distortion correction of a plane wave beam by the double-pass of the beam through the distorfer after reflecting on a phase conjugation mirror (PCM). \( E_0(t) \) is the incident beam,
The realisation of a practical PCM relies on the nonlinear response of a wide range of materials to sufficiently coherent radiation. The nonlinear behaviour of such media can couple together optical waves significant resulting in the transfer of energy. The associated nonlinear polarisation can even generate new waves which, under some circumstances, exhibit the required characteristics of the phase conjugate signal, as it was discussed in the previous section of this chapter.

When the polarisation, \( \bar{P}_i(\omega) \), due to the nonlinearity \( \chi^{(3)}(\omega, \omega, \omega, -\omega) \) is inserted into the wave equation (3.32) and plane waves are assumed, the resulting spatial variation of the signal amplitude, \( A_4 \), is given by [21]

\[
\frac{dA_4}{dz} = i \frac{\omega}{2} \sqrt{\frac{\mu_0}{\epsilon}} \chi^{(3)} \left( |A_1|^2 + |A_2|^2 \right) A_4 + i \frac{\omega}{2} \sqrt{\frac{\mu_0}{\epsilon}} \chi^{(3)} A_1 A_2 A_3^* \tag{3.51}
\]

where it has been assumed that \( |A_1| = |A_2| \) and \( |A_1|^2 + |A_2|^2 \gg |A_3|^2 + |A_4|^2 \). The first term in the right-hand side of the above equation merely represents a phase change in \( A_4 \) if \( \chi^{(3)} \) is real, or an amplitude change (gain/absorption) if \( \chi^{(3)} \) is imaginary. It can be, therefore, be factored out by defining \( A_4 \) as follows

\[
A_4(z) = A'_4(z) e^{i \frac{\omega}{2} \sqrt{\frac{\mu_0}{\epsilon}} \left( |A_1|^2 + |A_2|^2 \right)} \tag{3.52}
\]

resulting in the equation
\[
\frac{dA'_4}{dz} = K A'^*_3
\]  
(3.53)

where \( K \) is the complex FWM coupling coefficient given by

\[
K = i \frac{\omega}{2} \sqrt{\frac{\mu_0}{\varepsilon}} \chi^{(3)} A_1 A_2
\]  
(3.54)

Equation (3.53) and the corresponding equation for the probe \( A'_3 \), have been solved \[21\] under the assumption of undepleted pumps (constant pump beam energy) and shown to give an output signal of the form \( A_4(0) = r_c A'_3(0) \), where \( r_c \) is a complex constant representing the amplitude reflectivity of the PCM. This is exactly the form required of the phase conjugate as defined before in this section of the chapter. The amplitude reflectivity \( r_c \) depends on the strength of the nonlinearity \( \chi^{(3)} \), the two pump waves, the interaction length of the medium and on the phase mismatch, \( \Delta k \), if there is a small frequency detuning. Therefore DFWM can form the basis of a practical PCM with an intensity reflectivity given by

\[
R = \left| \frac{A_4(0)}{A'_3(0)} \right|^2 = |r_c|^2
\]  
(3.55)
3.8 References


697 (1997).
Chapter 4

Continuous-wave diode-pumped Nd:YVO4 holographic laser oscillator

4.1 Introduction

Nowadays good spatial quality and high-power beams are needed for laser systems to fulfill the requirements of many applications; however the main problem of the high power laser systems is the heating of its active medium caused by the high pumping powers used. In the high-power solid-state laser systems this heating causes phase distortions as well as depolarization [1]. As a solution to this problem, the optical phase conjugation technique (OPC) [2] has been used in order to correct for intracavity phase distortions by means of the generation of dynamic gain gratings in the active medium [3]. This nonlinear technique has been applied in the construction of several self-adaptive pulsed solid-state laser systems like flashlamp pumped Nd:YAG lasers [4], diode-pumped Neodymium: Yttrium Orthovanadate (Nd:YVO₄) lasers [5], and lasers-pumped Ti:sapphire lasers [6].

The self-adaptive lasers are constructed by using the four-wave mixing OPC technique in order to induce the formation of a gain grating in an active medium. The gain grating is produced by the gain saturation in the amplifier laser medium obtained through the interference of two coherent intersecting beams. In this way, the self-adaptive laser is created via the diffraction produced by the gain grating when the amplifier medium is
located in a self-intersecting geometry, see Fig. 4.1. This gain grating is a volume hologram that codifies the distortions from the oscillator loop, including those caused by the amplifier medium itself, and in this way allows the correction of the phase distortions. Because in the self-adaptive laser systems the formation of the loop oscillator is achieved by the creation of a volume hologram resulting from the induced gain grating, those lasers have been referred to as holographic laser oscillators (HLO) [7].
Although the high-gain requirements of this technique made its implementation to continuous-wave lasers to be more difficult to achieve, recently it has been shown that it is possible to obtain an efficient OPC in a continuous-wave side-pumped laser amplifier used in a four-wave mixing configuration [8]. Here we report on the application of the grain grating technique to the construction of a continuous-wave diode-pumped Nd:YVO₄ HLO.

Figure 4.2 Transmission of the HLO as a function of the NRTE wave-plate angle.
4.2 The experimental holographic laser oscillator

In the self-intersecting configuration shown in Fig. 4.1, an injected beam propagating in the anti-clockwise direction passes through the amplifier and is redirected to self-intersect in the amplifier again. The interference pattern generated by this self-intersection of the injected beam in the amplifier produces the modulation of the population inversion inside the active medium. This modulation forms a volume gain grating which keeps information of any phase distortion suffered by the injected beam along its whole propagation in the closed loop oscillator, which is formed by the Bragg-matched diffraction of the gain grating light. This allows the oscillation of the light in the opposite direction to the injected beam, the clockwise direction in Fig. 4.1, when the loop gain reaches its threshold level.

The non-reciprocal transmission element (NRTE), formed by a half-wave plate and a Faraday rotator located between a pair of polarizers, increases the operation efficiency since it maximizes the gain grating modulation and allows the unidirectional oscillation in the clockwise direction due to its different transmissions for the anti-clockwise and clockwise directions [9]. The laser radiation propagating in the clockwise direction can form a self-consistent spatial mode which is the spatial phase conjugate of the injected beam [10]. This conjugated beam effectively corrects for the aberrations inside the loop oscillator and maintains the high spatial quality of the output beam [7].

In the experimental HLO shown in Fig. 4.1, the used amplifier was a Nd:YVO₄ crystal at 1.1 atm % with dimensions of (20mm x 5mm x 1mm) from Casix Inc. The (5mm x 1mm) surfaces were anti-reflection coated for the lasing wavelength, 1064nm; and the
pump surface, the (20mm x 1mm) surface, was anti-reflection coated for the pumping wavelength, 808nm. The self-lasing inside the crystal was highly suppressed by cutting the (5mm x 1mm) surfaces of the crystal at an angle of \( \approx 2^\circ \) with respect to the normal of the pumped surface.

The pumping source was a 25W continuous-wave fast-diverging axis collimated diode bar from Coherent Inc., which was focused in the Nd:YVO\(_4\) crystal by a cylinder lens with focal length of 12.7mm. Due to the birrefringence of the Nd:YVO\(_4\), the polarization of the light emitted by the pumping diode had to be rotated by means of a half-wave plate to be parallel to the c-axis of the crystal in order to access its high absorption cross section for the pumping wavelength [11].

When the Nd:YVO\(_4\) amplifier is pumped, the non-uniform exponential decay of the gain region in function of the deep inside the crystal can be highly compensated by angling the beam being amplified. The inclined incidence on the amplifier allows the beam being amplified to experience a total internal reflection in the pumped surface of the crystal. This total internal reflection allows the averaging of the transverse gain variation along the spatial profile of the beam being amplified. Such a configuration, known as bouncing geometry, provides with the high gain needed to obtain an efficient operation of the HLO [11]; and it has been shown that the observed small signal gain is a function of the total internal reflection angle of the beam being amplified [12]. Because the gain region is narrow in the transverse plane, in order to access it, it was necessary to use 10cm focal length spherical focusing and recollimating lenses at each side of the amplifier.
Figure 4.3 NRTE

The right orientation of the NRTE half-wave plate allows the reduction of its transmission in the anti-clockwise direction (injected beam) in order to optimize the gain grating formation, while this transmission for the clockwise direction (output beam) is increased in order to produce an efficient laser oscillation. The HLO anti-clockwise transmission, \( T^+ \), was directly measured by varying the NRTE half-wave plate angle, \( \theta \), then by adjusting this angle to allow the maximum anti-clockwise transmission (\( \theta = 45^\circ \)) it was possible to determine \( T_0 = 56.8\% \) in:

\[
T^+ = T_0 \sin^2 2\theta
\]  

(4.1)
therefore the HLO clockwise transmission, $T_\phi$, could be calculated using:

$$T_\phi = T_0 \cos^2 \theta$$

(4.2)

Figure 4.4 Output beam TEM_{00} spatial mode

and is shown together with the HLO anti-clockwise transmission in Fig. 4.5 as a function of the NRTE half-wave plate angle.

The injection source for the HLO was an external continuous-wave Nd:YVO$_4$ laser from Crystal Lasers Inc., which spectral characteristics of single-mode and frequency
were inherited to the HLO output beam. This injection laser was also used to measure the single pass gain of the amplifier.

Figure 4.5 Output power of the HLO as a function of the injection beam input power.
4.3 Results and discussion

The highest output power obtained for the experimental HLO was \(\approx 5.7\)W with an internal incidence angle of \(\approx 2^\circ\) with respect to the pumped surface; a separation angle between the self-intersected beams of \(\approx 1^\circ\), and an injection power of \(\approx 25.5\)mW.

Fig. 4.4 shows the output TEM\(_{00}\) spatial mode for the maximum obtained output power; which is elliptical due to the asymmetry of the gain region. This output beam was extracted from the HLO by means of an optical isolator located between the HLO and the injection laser; and recorded by using a CCD camera and a beam analyzer from Spiricon Inc. The self-adaptive capability of the oscillator became evident since the HLO was able to produce such an spatial high quality mode like the one shown in Fig.4.4.

In Fig. 4.5, we show the experimental measurements of the output power of the HLO as a function of the injection beam input power, for an anti-clockwise NRTE transmission of \(\approx 1.5\%\). In this Fig. 4.5 it is possible to observe that there exists an optimum injection beam power for a given small signal gain of the amplifier.
Figure 4.6 Single pass gain for the HLO.

Measurements of the oscillating beam in the plane of the first focusing lens, the left-hand side lens in Fig. 4.1, showed spot sizes of \( w_{xf} = 35 \, \mu m \) and \( w_{yf} = 30 \, \mu m \) for the horizontal and the vertical axes, respectively. With those spot sizes measurements at the focusing lens plane, it was possible to estimate the focused beam spot sizes the oscillating beam will have at some point near the centre of the amplifier, \( w_{x0} \) and \( w_{y0} \), using [13]:

\[
\omega_{x0,y0} \approx \frac{f \lambda}{\pi \omega_{xf,yf}}
\]  

(3)

where \( f = 10 \, cm \) is the focal length of the focusing lens and \( \lambda = 1.064 \times 10^{-4} \, cm \) is the oscillating beam wavelength. Therefore, the focused beam horizontal and vertical spot
sizes resulted to be \( w_{x0} \approx 93 \mu m \) and \( w_{y0} \approx 108 \mu m \), and those calculations allowed us to approximate the focused beam transverse area as \( A = \pi w_{x0} w_{y0} 3.15 \times 10^{-4} \text{ cm}^2 \).

This estimation of the focused beam transverse area was used together with the maximum measured output power, \( P_{\text{out}} \approx 5.7 \text{W} \), in equation:

\[
I_{st} = \frac{P_{\text{max}}}{A}
\]  

(4)

in order to calculate the stored intensity as \( I_{st} \approx 18 \text{kW/cm}^2 \); which then could be used to obtain the value of the Nd:YVO$_4$ absorption coefficient-length product, \( \alpha_0 L \), from relation:

\[
I_{st} = 2\alpha_0 I_{\text{sat}}
\]  

(5)

where \( I_{\text{sat}} \approx 1 \text{kW/cm}^2 \) is the saturation intensity of the Nd:YVO$_4$.

Finally, this value of \( \alpha_0 L \) was used in order to estimate a small signal gain coefficient from:

\[
G_0 = \exp (2\alpha_0 L)
\]  

(6)
which resulted to be $G_0 \approx \exp(18) = 7 \times 10^7$; being consistent with our experimental measurements for the single pass gain and the phase-conjugate reflectivity shown in Figs. 4.6 and 4.7, respectively, after considering that some recombination processes take place in the Nd:YVO$_4$ amplifier. Perhaps the most important of those being the Auger upconversion recombination process [14]. A simple theoretical model was used to show the Auger upconversion recombination process effect in the small signal gain of the oscillator and the results are also shown in Fig. 4.5.
4.4 Gain volume overlapping optimization in a Nd:YVO$_4$ holographic laser oscillator

The aim of this part is to show that we can optimize the overlap area in Nd:YVO$_4$ crystal in order to increment the gain region efficiency and the grating diffraction in the crystal.

Next fig. 4.7 shows the real spot size calculated whit 4x4 matrices in a round trip inside de resonator and is cooperated with the experimental data.

Fig. 4.7 Experimental data and theoretical model
In the next figure it is possible to observe that the mode spot size inside of the crystal has a “small” difference between the spot size at first pass and the spot size at the second pass.

Fig. 4.8. Overlap area inside crystal in the HLO

Using the matrix method for Gaussian beams we can find the best position for each element intracavity.
Fig. 4.8 Overlap area optimized

Next figure shows the real lengths holographic oscillator analyzed in section 4.1
Fig 4.10 The best position.
4.4 Conclusions

As a conclusion, we have applied the gain grating technique to the construction of a continuous-wave diode-pumped injected holographic laser oscillator. The system provided an output power of up to \( \approx 5.7 \)W in a single longitudinal mode with the same frequency as that from the injected beam. The self-adaptive capability of the oscillator became evident since it was able to correct for the heat induced phase distortions in the amplifier and produce a high spatial quality TEM\textsubscript{00} output mode.
4.6 References


tion of a distortion-corrected Nd:YAG laser with a self-conjugating loop geometry I.E.E.E.


Chapter 5

Experimental Study of Second-Harmonic Generation by a Laser Pulse with Varying Direction of Polarization in a Type-II Synchronism Doubling Crystal

5.1 Introduction

Nowadays, a crystal Cr\textsuperscript{4+}:YAG is widely exploited as a saturable absorber (SA) for Q-switched solid-state lasers [1-12]. It has turned out that the direction of polarization of the generated giant pulse (GP) depends on the angular orientation of the Cr\textsuperscript{4+}:YAG crystal inside the cavity [3-11] and that this direction can be controlled by a weak seeding signal [12]. Such pulses can be used as a pump in type-II second-harmonic generation (SHG) when one may need a harmonic-pulse shortening and / or the possibility to transform its shape. These features caused by a time variation of the direction of the GP polarization have been theoretically predicted in Ref.13. The aim of this study is to give an experimental verification of the model [13].

5.2 Preparation of pulses with varying azimuth of polarization

Let us describe an experimental setup allowing to get optical pulses with varying azimuth of polarization, giving preliminary a short theoretical sketch of its functioning
[13]. As it was mentioned above, it can be a neodymium laser passively Q-switched with a Cr$^{4+}$:YAG crystal (see Fig. 5.1.a). Such a laser allows one to obtain GPs of nanosecond range, at some conditions (see below) characterized by sweeping of the polarization azimuth due to the latent anisotropy of absorption in the Cr$^{4+}$:YAG SA appeared at the saturation stage [9].

The laser cavity is formed by two mirrors (1,2), with an active medium (AM (3), Nd$^{3+}$:YAG crystal), a SA (4) (Cr$^{4+}$:YAG crystal), and a glass plate as a partial polarizer (5) (PP) inside.

Figure 5.1 (a) Experimental setup, (b) Diagram of Cr$^{4+}$ centers in YAG crystal
The laser model

The laser dynamics is governed by a set of coupled rate equations [10]:

$$\frac{dF_a}{dt} = \frac{F_a}{t_R} \left[ 2\sigma_a N_d l_a - 2\sigma_s l_s \left( n_s^{(1)} \cos^2(\theta - \varphi) + n_s^{(2)} \sin^2(\theta - \varphi) \right) \right] \quad (5.1)$$

$$-\ln \left( \frac{1}{r} \right) - \alpha_x \cos^2 \varphi - \alpha_y \sin^2 \varphi \right]$$

$$\frac{dN_a}{dt} = -\gamma \sigma_a N_a F_a c \quad (5.2)$$

$$\frac{dn_s^{(1)}}{dt} = -\sigma_s n_s^{(1)} F_a c K \cos^2(\theta - \varphi) + \frac{n_s^0 - n_s^{(1)}}{\tau_s} \quad (5.3)$$

$$\frac{dn_s^{(2)}}{dt} = -\sigma_s n_s^{(2)} F_a c K \sin^2(\theta - \varphi) + \frac{n_s^0 - n_s^{(2)}}{\tau_s} \quad (5.4)$$

where $F_a$ is the average photon density inside the cavity in AM; $N_a$ is the population inversion in AM; $n_s^{(1)}$ and $n_s^{(2)}$ are the ground state populations of the Cr$^{4+}$ groups [100] and [010], respectively (see Fig.1.b); $n_s^0$ is the initial non-disturbed ground-state population of the Cr$^{4+}$ centers; $\sigma_a$ is the lasing cross-section of AM; $\sigma_s$ is the Cr$^{4+}$:YAG resonant absorption cross-section; $l_a$ and $l_s$ are the lengths of AM and Cr$^{4+}$:YAG SA, respectively; $K = S_a/S_s$ is the ratio of the transverse size of the laser beam in AM to that in SA; $r$ is the reflection coefficient of the output mirror (2); $\gamma$ is the AM population inversion reduction factor; $t_R$ is the round-trip time for the cavity of the length $L$ ($t_R = 2L/c$, where $c$ is the velocity of light); $\tau_s$ is the relaxation time of Cr$^{4+}$:YAG SA.
In order to describe the evolution of the state of polarization of the laser, assume that this is always an eigenstate corresponding to the minimum of the total intracavity losses.

One can express the evolution of $\varphi(t)$ as [9,12]:

$$
\varphi(t) = \frac{1}{2} \tan \left( \frac{\sin(2\theta)}{\cos(2\theta) - \frac{\alpha_y - \alpha_x}{2l_s \sigma_s(n_s^{(1)}(t) - n_s^{(2)}(t))}} \right)
$$

(5.5)

where $(\alpha_y - \alpha_x)$ is the PP partial losses difference, being determined from the Fresnel formulas for a tilted (at the angle $\beta$) glass plate and $\theta$ is the angle between the group of $\text{Cr}^{4+}$ centers of [100] orientation and the axis X giving the direction of minimum non-saturated losses $(\alpha_x)$ of the cavity.

![Fig. 5.2 a](image-url)
Fig. 5.2b
Fig. 5.2 Simulated dynamics of laser: Dependences vs time of intracavity intensity (a), polarization azimuth rotation (b); number of Cr$^{4+}$ centers of orthogonal (1–1' and 2–2') orientations in the ground state (c). Curves 1, 1' correspond to orientation of Cr$^{4+}$:YAG "0°", curves 2, 2' – to its orientation "40°".

It can be seen from Eqs. (5.1-5.5) that the direction of polarization of the GP evolves nonlinearly owing to the interplay between the linear anisotropy of the cavity which is
due to the PP presence and the self-induced anisotropy of the SA, which is caused by the \( \text{Cr}^{4+}:\text{YAG} \) crystal presence.

Fig.5.2,3 give the numerical results of the model, which is experimentally studied in Section II.2 and whose parameters are collected in Table 1.

In Fig.5.2 the temporal characteristics of the modeled laser are shown for the two cases: operation with stable (curves 1) and rotated (curves 2) azimuth of polarization. Fig.5.2,a represents intensity of the GP generated; Fig.2,b shows the dynamics of azimuth of polarization; and Fig.5.2,c,d demonstrate how change in time the populations \( n_s^{(1)} \) (curves 1,2) and \( n_s^{(2)} \) (curves 1',2') of the orthogonal groups of the \( \text{Cr}^{4+} \) centers in the \( \text{Cr}^{4+}:\text{YAG} \) SA. It is seen that a simple change of the \( \text{Cr}^{4+}:\text{YAG} \) angular position in the cavity results in enforcing the laser to oscillate in quite different polarization regimes; this difference is connected with bleaching of only one group of the \( \text{Cr}^{4+} \) centers (in the case of operation in the mode with fixed polarization, Fig.5.2,c) and virtually in-phase bleaching of both the orthogonal \( \text{Cr}^{4+} \) centers (in the case of nonlinearly varying azimuth of polarization, Fig.5.2,d). Note that despite azimuth of polarization has essentially differed behavior in these two circumstances (compare curves 1 and 2 in Fig.5.2,b), the GPs oscillated are close each to other in amplitude, but slightly different in width (the pulse with rotated polarization is wider in time scale, Fig.5.2,a). Let us mention that similar features are observed experimentally (see below).

Fig. 5.3 shows three particular snapshots of what is the result of action of a polarizer (say, Glan prism (6), Fig.1,a) set at the output of the laser, when polarization azimuth experinces rotation in time. It is seen that, when the polarizer is set close to the crossed position with respect to the instantaneous polarization in the pulse maximum, a dip in the output pulse appears. It is characteristic that the resultant pulse shape
can be controlled by slight rotation of the polarizer (compare Fig.3,a-c). Let us mention that such a procedure is the most simple way to monitor nonlinear dynamics of the laser state of polarization.

Fig 5.3 a

Fig 5.3 b
Figure 5.3 Modeled pulses at Gland Prism output (orientation of \( Cr^{4+}:YAG \) “40°” (a) \( \Psi = 70° \), (b) \( \Psi = 75° \) and (c) \( \Psi = 80° \))

Table 1

<table>
<thead>
<tr>
<th>AM:</th>
<th>SA:</th>
<th>Cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nd(^{3+}):YAG</td>
<td>Cr(^{4+}):YAG</td>
<td>L = 32 cm</td>
</tr>
<tr>
<td>( \sigma_a = 6.5 \times 10^{-19} ) cm(^2)</td>
<td>( \sigma_s = 5.6 \times 10^{-18} ) cm(^2)</td>
<td>r = 0.6</td>
</tr>
<tr>
<td>( l_a = 5 ) cm</td>
<td>( l_s = 0.3 ) cm</td>
<td>( \beta = 50° )</td>
</tr>
<tr>
<td>( h\nu = 1.85 \times 10^{-19} ) J</td>
<td>( T_{in} = 30% ), ( T_{fin} = 90% )</td>
<td>Spot size ( S_a = 0.089 ) cm(^2)</td>
</tr>
<tr>
<td>( \gamma = 0.6 )</td>
<td>( \tau_s = 3.0 \times 10^{-6} ) s</td>
<td>K = 2.0</td>
</tr>
</tbody>
</table>
5.2.1 The experimental realization

The laser studied was described above. Let us make some additional remarks about the setup employed. Together with the AM, SA, and PP (with parameters listed in Table 1), the cavity contained a diaphragm for selecting the TEM\textsubscript{00} mode; this was essentially important, since even slight “dope” of the highest transversal modes in radiation complicate polarization behavior of the laser. The laser was pumped by a flashlamp and operated at the repetition rate of less than 1 Hz in order to decrease possible influence on the polarization state of thermally induced birefringence in AM and SA. The characteristic energy of GPs was \(~1\) mJ. Control over the polarization state of the laser was done by a proper orientation of the PP (\(\beta = 50^\circ\)) and rotating the Cr\textsuperscript{4+}:YAG switch relatively direction of minimum linear losses of the cavity determined by the glass plate. The angle of the Cr\textsuperscript{4+}:YAG SA orientation in the cavity \(\theta\) was measured as it is shown in Fig.5.1,b.

Fig.5.4 gives the snapshots of a GP at the laser output (before the extracavity polarizer) for the two specific orientations of the Cr\textsuperscript{4+}:YAG SA - 0\(^\circ\) (Fig.5.4,a) and 40\(^\circ\) (Fig.5.4,b). In the first case, as the theory predicts, the fixed azimuth of polarization has to be observed, while in the second one – the nonlinearly rotated one. The first observation deduced from Fig.5 is that in the “0\(^\circ\)” position of the Cr\textsuperscript{4+}:YAG crystal the pulse is slightly wider than in the “40\(^\circ\)” one (compare with the theoretical pulses given in Fig.5.2,a). Only this fact allows to suppose that serious differences exist in the laser dynamics.
To verify whether or not these changes are connected with the polarization dynamics, we analyzed the radiation after the extracavity polarizer (6). The correspondent snapshots are given in Fig.5.5,a-c for various orientations of the polarizer (given by the angle \( \psi \)), when the Cr\(^{4+}\):YAG SA is set in the position "40°". It is clearly seen that the oscillograms resemble the theoretical ones (see Fig.5.3,a-c), i.e. the characteristic dips.
in the output pulse intensity are observed, which position in time is monitored by slight rotation of the polarizer. Meanwhile, when the Cr$^{4+}$:YAG SA is set in position "0°", quite another situation is observed: In the crossed position of the polarizer we measured virtually zero background. Thus, it is evident that the laser studied possesses to oscillate either stable state of polarization (position "0°" of the Cr$^{4+}$:YAG SA) or the state with azimuth nonlinearly changing in time (position "40°" of the Cr$^{4+}$:YAG SA). It is evident that all these transformations in the polarization dynamics are connected with the particular orientation of the Cr$^{4+}$:YAG SA only, testifying for the findings of the theoretical model of the laser.

Fig 5.5 a

Fig 5.5 b
Figure 5.5 Experimental pulses at Glanl Prism output (Orientation of Cr^4+:YAG ≈40°): (a) Ψ=72°, (b) Ψ =76° and (c) Ψ=79° (20 ns/div).

5.3 SHG of pulses with varying azimuth of polarization

Let us now consider the case where the polarizer (6) at the laser output is replaced by a doubling, say, a $\chi^{(2)}$-crystal with type-II synchronism (7) (see Fig.5.1,a). Like in Section 5.2, let us stop primarily on the theoretical description of the doubling process, when a GP with varying azimuth of polarization is treated as a pump source for a traveling-wave SHG. The experimental realization of the SHG phenomenon in a KTP crystal will be further discussed.

5.3.1. The doubling process modeling

SHG in a doubling crystal with type-II synchronism is simulated by the following couple of equations [13]:

$$\frac{dA_p^{(1)}}{d\xi} = -A_p^{(2)} A_s \sin \eta \quad (5.6)$$
\[
\frac{dA_p^{(2)}}{d\xi} = -A_p^{(1)}A_s\sin\eta 
\] (5.7)

\[
\frac{dA_s}{d\xi} = A_p^{(1)}A_p^{(2)}\sin\eta 
\] (5.8)

\[
\frac{d\eta}{d\xi} = \delta - \left( \frac{A_p^{(2)}A_s}{A_p^{(1)}} + \frac{A_p^{(1)}A_s}{A_p^{(2)}} - \frac{A_p^{(1)}A_p^{(2)}}{A_s} \right)\cos\eta ,
\] (5.9)

which uses the solution of Eqs. (5.1-5.5) as a boundary condition and where \( A_p^{(1)}, A_p^{(2)} \) and \( A_s \) are the normalized real amplitudes of the pump and SH waves, respectively; \( \eta \) is the relative phase between the harmonic and the pump waves \( (\eta = \eta_s - 2\eta_p) \); \( \delta \) is the normalized phase mismatch; and the nonlinear coupling is chosen as a normalization parameter \( \xi \) [14,15]. The set of Eqs. (5.6-5.9) are derived under assumptions of the steady-state interaction between the fundamental wave and SH in a transparent quadratic material. The silent advantage of these approximations is that they are well suited for the nanosecond pulses. It is also worth to notice that under our approximations the amplitude of the fundamental wave relates to the intensity of the pulse \( I \), polarization azimuth \( \varphi \), and the orientation of the doubling crystal \( \psi \) as:

\[
A_p^{(1)} \propto \sqrt{I}\cos(\varphi - \psi) ;
\] (5.10a)

\[
A_p^{(2)} \propto \sqrt{I}\sin(\varphi - \psi) .
\] (5.10b)

In our simulations we also assume the zero input at the harmonic frequency and permit \( \eta_0 = 0; \delta_0 = 0.01. \)

Fig.5.6,a,b gives the results of numerical calculations of Eqs. (5.1-5.10) when the incidence pump pulses are, respectively, those described by curves 5.1 and 5.2 in Fig.5.2,a (fixed polarization, orientation "0" of the Cr\(^{4+}\):YAG SA and nonlinearly
rotating polarization, its orientation "40°"). Both snapshots are calculated for the SHG crystal orientation corresponding to maximum efficiency of transformation of the pump in SH. It is seen that considerable shortening of the SH pulse is observed for the pump with changing azimuth of polarization (Fig. 5.6, b, curve 1) comparing the case of fixed polarization of the incidence (Fig. 5.6, a, curve 1).

Figure 5.6 Modeling SGH (a) with fixed, orientation of Cr³⁺:YAG "0°", (b) nonlinearly rotating, orientation of Cr³⁺:YAG "40°"; azimuth of polarization. Curves 1- SH output pulse (λ = 0.53 μm); Curves 2- fundamental input pulse (λ = 1.06 μm).
It is also interesting to analyze SHG, when the doubling crystal is tuned close to the minimum transformation efficiency of the pump in SH. In these circumstances, calculations give the resultant envelopes of the SH pulse shown in Fig. 5.7a-c. It is remarkable that the snapshots of the SH pulses resemble those appeared when a polarizer is set at the laser output (compare Fig. 5.7a-c and Fig. 5.3a-c). This fact is not surprising, since the both extracavity elements (Glan prism as a polarizer and a SH crystal) play the same role of an analyzer of the state of polarization [16]. Note only that in the last case the output SH pulse width is less than that of the fundamental pulse at the Glan prism output.

Fig 5.7a

Fig 5.7b
Figure 5.7 Modeled pulses at SH crystal output (orientation of Cr\textsuperscript{4+}:YAG "40°") : (a) $\Psi = 70^\circ$, (b) $\Psi = 75^\circ$ and (c) $\Psi = 80^\circ$

Let us stop finally on what are the features of efficiency of transformation in SH and the SH pulses’ duration (measured at a half of amplitude) in the cases of fixed and rotating azimuth of polarization of the pump. Fig. 5.8 gives the correspondent dependences versus orientation (angle $\psi$) of the doubling crystal. From these graphs one can conclude on virtually the same efficiency of SHG process in both situations (compare dependences in Fig.5.8,a and Fig.5.8,c). (Of course, it concerns the peak intensity of SH signals only, whereas the resultant SH pulse energy for nonlinearly rotated polarization of the input is considerably (by a factor of $\sim 2.5 - 3$) less than for the launched pulse with fixed polarization, see Fig.5.6). Let us finally note the possibility to evaluate the angle of rotation of the polarization azimuth using a doubling crystal as an analyzer. The main fact is, however, that in the case of varying polarization, at some orientation of the doubling crystal, one can get SH pulses with essentially shortened duration comparing the case of fixed polarization of the pump (compare dependences in Fig.5.8,b and Fig.5.8,d).
Figure 5.8 Theoretical dependences of SH pulse peak intensity (a,c) and pulse duration (b,d) calculated for the cases of fixed (a,d) and nonlinearly rotation (a,d) azimuth of polarization.
5.3.2. The experimental realization of SHG of pulses with varying polarization

SHG of the pulses launched from the master oscillator was experimentally realized with a KTP crystal cut at type-II synchronism. The crystal (7) was set at the place of the Glan prism polarizer (6) (see Fig.5.1,a). By its rotation about the optical axis (angle $\psi$), we could change efficiency of the doubling process. As it was shown above in modeling of the SHG process, one might expect strong differences in SHG depending on whether or not an azimuth of polarization experiences nonlinear rotation during the GP.

Fig 5.9 a

Fig 5.9 b
Fig 5.9 c

Figure 5.9 Experimental pulses of fundamental input ($\lambda=1.06 \ \mu m$) pulse (a) and SH output ($\lambda=0.53 \ \mu m$) pulses at minimum (b, $\Psi=75^\circ$) and maximum (c, $\Psi=25^\circ$) efficiency of doubling (20 ns/div).

Fig 5.10 a
Fig 5. 10 b

Figure 5.10 Experimental dependences of SH pulse peak intensity (a) and pulse duration (b) for the cases of fixed (curves 1) and nonlinearly rotating (curves 2) azimuth of polarization.

Fig.5.9,5.10 give the experimental pattern of the SHG process in the case under study. It is seen from Fig.5.9 that, comparing the incidence fundamental 1.06 μm pulse with rotating azimuth of polarization (Fig.5.9,a), the SH pulse - either by positioning the KTP crystal close to minimum of SHG efficiency (Fig.5.9,b), or close to its maximum (Fig.5.9,c) - has less width. Meanwhile, in the minimum of SHG transformation efficiency, the SH pulse resembles that exiting the Glan prism polarizer (Fig.5.5). Note that all the features figured out experimentally agree quite well with those calculated numerically (compare, for instance, Fig.5.9,b and Fig.5.7,a). The last fact is not surprising, since, as it was mentioned above, a doubling crystal plays a role of a polarization analyzer.
Finally, Fig. 5.10 shows the dependences of SH pulse peak intensity (Fig. 5.10,a) and the resultant SH pulse width measured at a half of amplitude (Fig. 5.10,b) on the KTP crystal angular orientation $\Psi$. Curves 1 and 2 give, respectively, the dependences for the input pulse with fixed and nonlinearly rotating azimuth of polarization. The main observations, which might be deduced from this set of curves, is that:

1. There is considerable shortening of the SH pulses, when the incidence ones possess of polarization rotation comparing the case of no polarization rotation (compare curves 1 and 2 in Fig. 5.10,b).

2. Doubling crystal might provide a fairly good tool for analysis of the state of polarization of a pulsed light (note that this fact has been analyzed in Ref. 16 in the steady-state approximation). For example, one can easily find from the curves 1 and 2 in Fig. 5.10,a that the maximum polarization azimuth rotation during the GP is equal to $\sim 35^\circ$ (compare this value with the data for polarization azimuth scenarios obtained in the numerical calculations, Fig. 5.2,b), but the novel fact is that the direction of the rotation manifests itself in pronounced asymmetry of the curves 2 in Fig. 5.10,b, making possible to determine not only the magnitude of the rotation, but its direction as well.

5.4 Conclusions

In summary, we have studied experimentally the type-II SHG process of the laser pulse with direction of polarization experiencing nonlinear rotation in time. It has been found out that the shape of a generated SH pulse can be controlled by a simple angular displacement of the Cr$^{3+}$:YAG SA in the master oscillator cavity. The harmonic pulse shortening has been observed in these circumstances comparing the case of a launched
fundamental pulse with a fixed azimuth of polarization. It has been demonstrated that SH crystals may be used for estimation of the polarization dynamics of neodymium lasers operating in the passive Q-switch mode. The experiments have clearly confirmed the correspondent model of the phenomenon [13].
5.5 References


Chapter 6

Conclusions

As a conclusion, a spatial mode spot size study of a diode-side pumped Nd:YVO₄ laser resonator has been presented. The spatial study was done in order to find the best possible stable cavity configuration. The stability analysis of the resonator showed five possible stable configurations for the laser resonator. The mode spot size behaviors for the sagittal and tangential planes along the resonator were obtained for each of the five possible stable configurations found. From the analysis of the mode spot size behavior on both planes, it was possible to determine the best option to build the diode-side-pumped Nd:YVO₄ laser resonator. This study has been shown to be useful in the optimization of holographic resonator based in Nd:YVO₄ using similar pumping configuration.

An experimental setup using the gain grating technique to the construction of a continuous-wave diode-pumped injected holographic laser oscillator was studied. The system provided an output power of up to ≈5.7W in a single longitudinal mode with the same frequency as that from the injected beam. The self-adaptive capability of the oscillator became evident since it was able to correct for the heat induced phase distortions in the amplifier and produce a high spatial quality TEM₀₀ output mode.

And finally, pulsed systems were also studied in particular, the type-II SHG process of the laser pulse with direction of polarization experiencing nonlinear rotation in time. It has been found out that the shape of a generated SH pulse can be controlled by a simple angular
displacement of the Cr\textsuperscript{4+}:YAG SA in the master oscillator cavity. The harmonic pulse shortening has been observed in these circumstances comparing the case of a launched fundamental pulse with a fixed azimuth of polarization. It has been demonstrated that SH crystals may be used for estimation of the polarization dynamics of neodymium lasers operating in the passive Q-switch mode. The experiments have clearly confirmed the correspondent model of the phenomenon.