Phase-locking phenomenon in a semiconductor laser with external cavities

F. R. Ruiz-Oliveras and A. N. Pisarchik

Centro de Investigaciones en Optica, Loma del Bosque115,
Lomas del Campestre, 37150 Leon, Guanajuato, Mexico
flavio@cio.mx, apisarch@cio.mx

Abstract: Phase-locked solutions are found numerically in a semiconductor laser with one and two external cavities. Different periodic, quasiperiodic, chaotic, and steady-state regimes form Arnold's tongues in bi-dimensional parameter spaces of the length and feedback strengths of the external cavities and the pump parameter. This rich structure gives additional possibility for controlling complex dynamics and chaos in a semiconductor laser with external cavities by properly adjusting their lengths and feedback strengths.

OCIS codes: (140.5960) Semiconductor lasers; (190.3100) Instabilities and chaos

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1. Introduction

The dynamics of a semiconductor laser subjected to external optical feedback has been studied by many researchers, since this laser has many applications in optical communications, interferometric sensors, etc. (see, for example, [1] and references therein). For moderate and strong feedback strengths this laser can display a very rich dynamical behavior, from periodic and quasiperiodic oscillations to chaos and coherence collapse [2,3]. When the laser injection current is close to the solitary laser threshold, the laser operates in a stable steady-state regime. As the pump current is increased, intermittent drops of the laser intensity appear, that gives rise to low-frequency fluctuations (LFF). At higher currents, the laser optical bandwidth broadens that is known as coherence collapse [4,5].

The dynamical behavior of a semiconductor laser with a single feedback was studied extensively and the basic mechanism for LFF is well understood [6,7]. The possibility for controlling LFF in a semiconductor laser with an external cavity by using a second delayed optical feedback was initially deduced by Liu and Ohtsubo [8]. Later their idea was developed by Rogister *et al.* [9,10], who realized the suppression of antimodes, responsible for LFF crises by properly adjusting the second feedback strength.

Phase locking is one of the general phenomena in physics. This phenomenon manifests itself when two oscillators are coupled together and entrain each other to some rational multiple of their natural frequencies, so that their combined motion becomes periodic with a common frequency. In other cases, their combined motion has two independent frequencies resulting in quasiperiodicity and chaos. These locked and unlocked regions typically form a sophisticated structure in parameter space. The locked regions are called *Arnold tongues* [11]. Recently Mendez *et al.* [12] have demonstrated experimentally that the frequency of LFF can be locked by external periodic modulation applied to the feedback strength of the external cavity. They have found different periodic and quasiperiodic regimes which formed Arnold tongues in the space of the frequency and amplitude of the external modulation.

In this paper we demonstrate a similar phase-locking phenomenon in a semiconductor laser without any external forcing. We demonstrate that in the laser with two external cavities the feedback time and the feedback strength of the second external cavity act in a similar manner as the period and amplitude of external modulation. The variation of the parameters of the second external cavity allows one to obtain different dynamical regimes of the laser operation without any modifications in the solitary laser with a single external cavity. Thus the dynamics of the semiconductor laser with two external cavities can be adequately controlled by adjusting properly both the length and the feedback strength of the external cavities.

2. Model equations

A single-mode semiconductor laser with delayed feedback is usually modeled by the Lang-Kobayashi rate equations [13]. Due to the infinite dimension of the system, an analytical study is very difficult. Therefore, we perform numerical simulations of a semiconductor laser with two external cavities, shown in Fig. 1, with modified equations similar to those explored previously by Sivapakrasam, *et al.* [14] and Carr [15].

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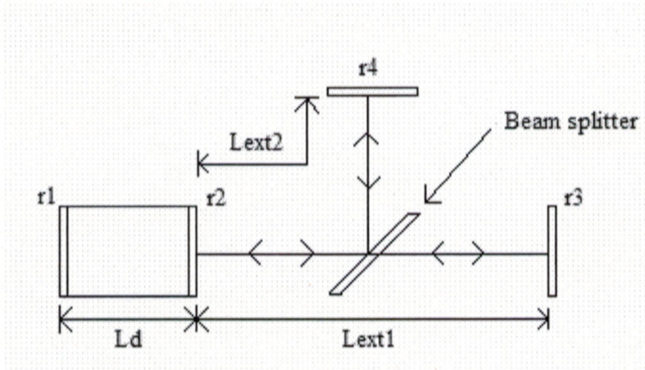

Fig. 1. Optical scheme of a semiconductor laser with two external cavities. Ld, Lext1, and Lext2 are the lengths of the internal and two external cavities, r1, r2 and r3, r4 are the reflectivities of the laser facets and the external mirrors, respectively.

For weak or moderate feedback the modified equations can be written as follows

$$d\dot{E}_0(t) = (1/2)G\left[\left(N(t)-N_{th}\right)\right]E_0(t)+\kappa_1 E_0(t-\tau_1)\cos\left[\psi_1+\varphi(t)-\varphi(t-\tau_1)\right]$$
$$+\kappa_2 E_0(t-\tau_2)\cos[\psi_2+\varphi(t)-\varphi(t-\tau_2)], \quad (1)$$

$$d\dot{\varphi}(t) = (\alpha/2)G\left[N(t)-N_{th}\right]E_0(t)-\kappa_1\left[E_0(t-\tau_1)/E_0(t)\right]\sin\left[\psi_1+\varphi(t)-\varphi(t-\tau_1)\right],$$
$$-\kappa_2\left[E_0(t-\tau_1)/E_0(t)\right]\sin[\psi_2+\varphi(t)-\varphi(t-\tau_2)], \quad (2)$$

$$d\dot{N}(t) = P-N(t)/\tau_s-G\left[N(t)-N_0\right]E_0^2(t). \quad (3)$$

Equations (1) – (3) are expressed with the variables of the amplitude $E_0(t)$ of the complex electric field $E(t) = E_0(t)\exp[i\omega_0 t + \varphi(t)]$ (ω_0 being the angular frequency of the solitary laser), the average carrier density in the active region $N(t)$ and the phase $\varphi(t)$. The carrier density at threshold $N_{th} = N_0 + (\tau_p G)^{-1}$, where N_0 is the carrier density at transparency, τ_p is the photon lifetime and G is the modal gain coefficient. The initial phases for the first and second external cavities $\psi_1 = \omega_0\tau_1$ and $\psi_2 = \omega_0\tau_2$, where τ_1 and τ_2 are the corresponding round trip times. The other parameters are τ_s is the carrier lifetime, P is the pumping term, α is the linewidth enhancement factor, and κ_1 and κ_2 are the feedback strengths for the first and second external cavities. The last two parameters are defined by the following formula: $\kappa_{1,2} = (1/\tau_0)(1 - r_{3,4})(r_{3,4}/r_2)^{1/2}$, where τ_0 is the round trip time for the internal cavity. In our simulations we use the following parameter values [16]: $\alpha = 3.5$, $G = 5\times10^{25}$ $\mathrm{m^3ns^{-1}}$, $\tau_s = 1$ ns, $\tau_p = 1$ ps, $P = 8\times10^{24}$ $\mathrm{m^{-3}}$, $N_{th} = 5\times10^{24}$ $\mathrm{m^{-3}}$ and $N_0 = 3\times10^{24}$ $\mathrm{m^{-3}}$. In all simulations performed with two external cavities, the parameters τ_1 and κ_1 of the first external cavity are fixed.

3. Results

First, we consider the semiconductor laser with one external cavity ($\kappa_2 = 0$) and then we show how the second external cavity affects laser dynamics. The analysis is performed with codimensional-one and codimensional-two bifurcation diagrams in the parameter spaces of the lengths and feedback strengths of the external cavities and the pump parameter.

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3.1 One external cavity

It is known that dynamics of a semiconductor laser with an external cavity exhibits a 2π-cyclic behavior with respect to the optical phase [16]. For the 837-nm wavelength semiconductor laser with one external cavity of 3.3-cm length and $\kappa_1 = 25$ ns^{-1}, the solution of the system Eqs. (1) – (3) is chaotic in the well-known form of LFF shown in Fig. 2.

In Fig. 3 we plot the state diagram of the laser with one external cavity in the bi-dimensional parameter space of the external cavity length $l_1 = c\tau_1/2$ (c being the speed of light) and the feedback strength. The diagram displays a striped structure, the alternation of steady-state, periodic, and chaotic regimes. In Fig. 3 and hereafter, the black dots indicate fix points or cw laser operation, the yellow, blue, and red are, respectively, the period-1, period-2, and period-3 regimes. The white regions indicate the quasiperiodic and chaotic regimes.

The analysis of the diagram shows that as either l_1 or κ_1 is changed, the laser undergoes different bifurcations: a Hopf bifurcation, a period-doubling bifurcation(s), and a torus bifurcation after which a quasi-periodic regime gives rise. The quasi-periodicity is converted to chaotic LFF terminated in crisis where a new cascade of bifurcations is initiated. In fact, the state diagram shown in Fig. 3 is the codimensional-two bifurcation diagram, where the boundaries between the black and yellow regions represent the Hopf bifurcation lines, the boundaries between the yellow and blue regions are period-doubling bifurcation lines, and the onsets of the white regions are the torus bifurcations. For very weak feedback strengths ($\kappa_1 <$ 0.4 ns^{-1}), the laser works in a cw (steady-state) regime for any length of the external cavity. Another interesting result is that for relatively strong feedback strengths, the width of the chaotic tongues increases with increasing the cavity length. Chaotic oscillations are not observed in the laser with a very short external cavity, i.e. the laser is always stable. Instead, the laser with a very long external cavity is always unstable.

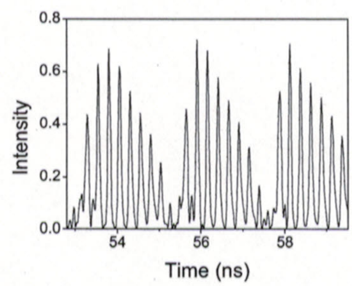

Fig. 2. Chaotic time series in semiconductor laser with one external cavity represented LFF.

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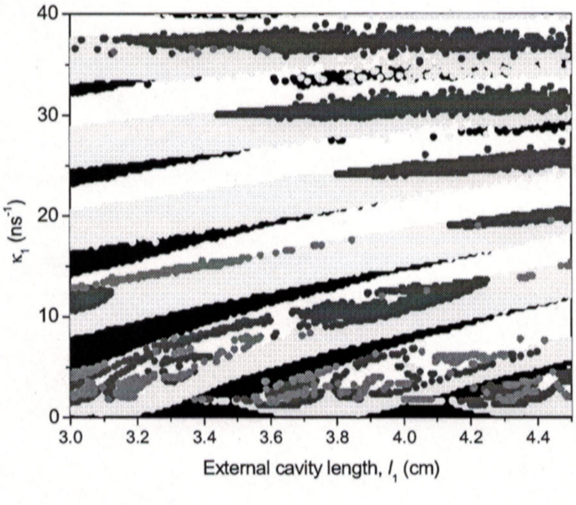

Fig. 3. State diagram of semiconductor laser with one external cavity in parameter space of its length and feedback strength.

The influence of the pumping on the dynamical behavior of the laser is illustrated in Figs. 4 and 5. The laser with a relatively short external cavity (l_1 < 1.7 cm) and a weak feedback strength (κ_1 < 1 ns^{-1}) works in a stationary (cw) regime independent on how much the pumping is. However, for a longer external cavity the laser dynamics is very rich. The state diagrams in both graphics form the structure of Arnold tongues which are distributed almost equidistantly along the abscissa axes. One can also see that while P is increased, the chaotic tongues become wider and finally at a very strong pump, chaos dominates over the other regimes.

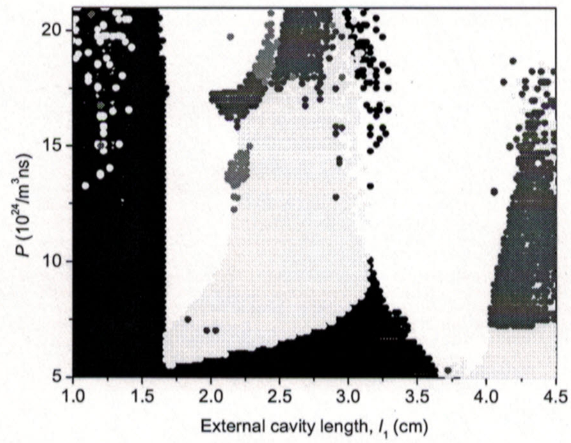

Fig. 4. State diagram of semiconductor laser with one external cavity in space of cavity length and pump parameter. κ_1 = 25 ns^{-1}.

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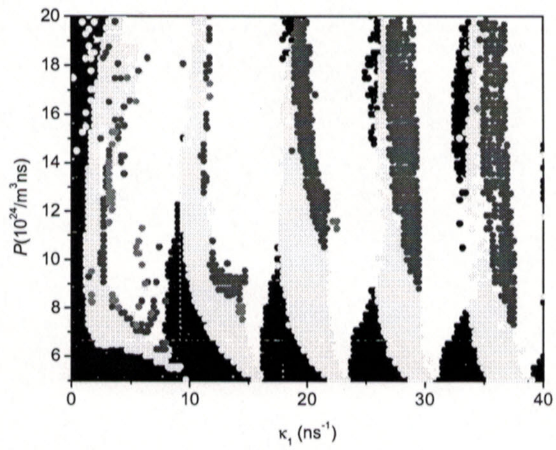

Fig. 5. State diagram of semiconductor laser with one external cavity in space of feedback strength and pump parameter. l_1 = 3.3 cm.

Arnold tongues are typically studied in the context of nonlinear circle maps with two degrees of freedom [17]. Here we have three degrees of freedom which are interacting with each other, and also infinite dimension of the time-delayed system. Moreover, the dynamics is continuous, not a discrete map. Therefore, our system is far more complex than the standard discrete 2D maps used to describe a phase-locked structure.

3.2 Two external cavities

The dynamics of the semiconductor laser with two external cavities is analyzed with codimensional-one and codimensional-two bifurcation diagrams in spaces of the laser parameters. Note, that the parameters of the first external cavity are hold to be constants, τ_1 = 0.22 ns and κ_1 = 25 ns^{-1}. One of the parameters of the second external cavity which can be easily controlled in experiments is its length $l_2 = c\tau_2/2$. First, we derive the codimensional-one bifurcation diagrams of the laser peak intensity with respect to $\tau_2/\tau_1 = l_2/l_1$ for different fixed values of the feedback strength and the pump parameter. One of such diagrams is shown in Fig. 6. One can see that the laser dynamics is very sophisticated and the diagram is not symmetric with respect to $l_2 = l_1$. By fixing τ_1, we solve Eqs. (1) – (3) for different τ_2. As the delay time of the second external cavity is changed, the laser undergoes different types of bifurcations (Hopf bifurcation, torus bifurcation, period-doubling bifurcation, and crisis). With increasing the delay time, the chaotic attractor, first, transforms to a torus and then the laser undergoes an inverse torus bifurcation resulting in period-1 oscillations which are terminated again in chaos.

We should note that in all our simulations the changes in the cavity length assume also the corresponding changes in the feedback phase. It is also possible that either the length or the phase is varied in the model independently. While the length is changed in large steps compared with the laser wavelength, the phase is, in general, only relevant when the cavity changes within a sub-wavelength precision. However, we do not include the analysis of the separate phase effect in the present paper, because the dynamics of a semiconductor laser with an external cavity under the phase change has been extensively studied and discussed in many papers (see, for example [16]). The bifurcation diagram of the laser intensity displays a 2π-cyclic behaviour with respect to the optical phase. The more important parameter for experimentalists is the cavity length because it is the real, easily measurable parameter, rather than the optical phase.

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In the inset diagram one can distinguish the regime of small-amplitude chaotic oscillations following by a torus bifurcation and chaos. As the cavity length is further increased, chaos is terminated in crisis and a limit cycle is created. This is a period-1 regime which disappears in an inverse Hopf bifurcation, where the laser emission becomes steadily stable. This steady-state regime or the fixed point can be interrupted by chaos which coexists with it.

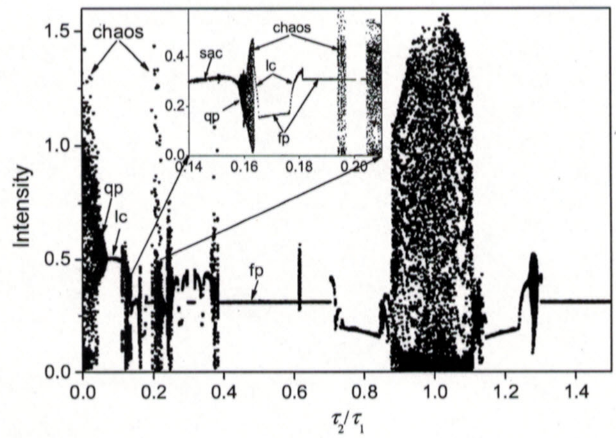

Fig. 6. Bifurcation diagram of laser peak intensity with τ_2/τ_1 as a control parameter. $P = 8 \times 10^{24}$ $m^{-3}ns^{-1}$ and $\kappa_2/\kappa_1 = 1$. The arrows indicate different dynamical regimes: quasiperiodicity (qp), limit cycle (lc), fixed point (fp), small-amplitude chaos (sac), and large-amplitude chaos (chaos).

Figures 7 and 8 display the state diagrams of the laser with two external cavities in the parameter spaces of the ratios of the round trip times and feedback strengths of the external cavities, τ_2/τ_1 and κ_2/κ_1, and pump parameter P. These codimensional-two bifurcation diagrams are constructed by analyzing the codimensional-one bifurcation diagrams, similar to that shown in Fig. 6, calculated for different fixed values of the feedback strength and the pump parameter. Although the parameters of the first external cavity are maintained to be constants, we plot these diagrams in the coordinates of the ratios of the cavity parameters for better understanding of the physical mechanism underlying the phase-locking phenomenon.

One can see in Fig. 7 that the Arnold tongues of the steady-state and periodic regimes alternate along both axes, i.e. there are cascades of bifurcations in both directions. This means that the phases of the two external cavities entrain each other, so that the laser motion becomes steadily stable or periodic at certain ratios of the cavity parameters. The minima of the tongues occur when the winding number $\tau_1/\tau_2 = p/q$ (both p and q being integers), i.e. at $\tau_2/\tau_1 = 1/1, 1/2, 1/4, 2/1, 3/2, \ldots$. It is particularly remarkable that the diagrams around $\tau_2/\tau_1 = 1$ are almost symmetrical. It is also possible for the tongues to overlap. In this case phase-locked solutions coexist at a point in the parameter space leading to multistability. The asymmetry in the diagrams appears just due to multistability. In the numerical simulations the phase-locked solution that is reached depends upon the basin structure for the coexisting attractors and the choice of initial conditions.

This basic scenario is quite common, and often manifests itself even when there is no obvious way to decompose the system into independent oscillators. The laser with two external cavities can be considered as a dynamical system composed by two oscillators coupled by the common active medium. Each of the oscillators has its own natural frequency defined by the phase of the external cavity. Thus, the phase locking phenomenon in such system is nothing but synchronization of two coupled autonomous oscillators. The second external cavity acts in a similar manner as an external periodic forcing applied to a self-

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oscillatory system [12], where the phase of the second cavity acts as the frequency of the external modulation while the feedback strength acts as the amplitude.

The phase-locking phenomenon can be used to control dynamical regimes and chaos in a semiconductor laser with external cavity. For example, LFF shown in Fig. 2 can be suppressed by adding a second external cavity and properly adjusting its length and feedback strength.

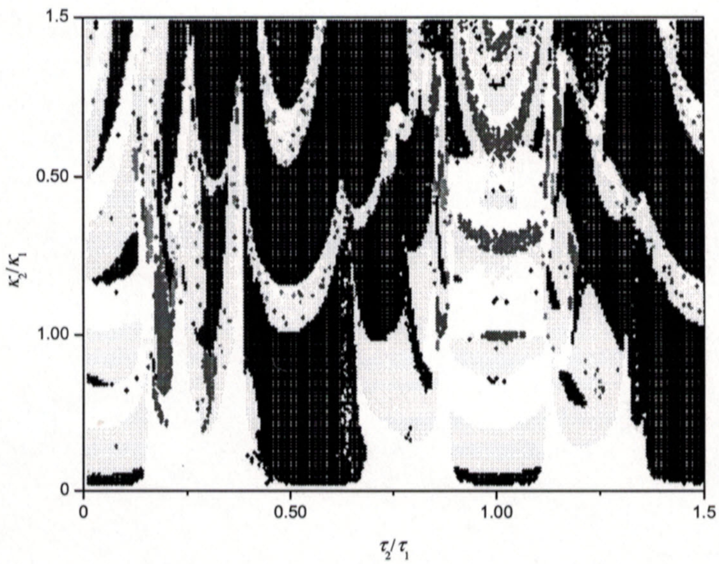

Fig. 7. State diagram of semiconductor laser with two external cavities in space of ratios of their round trip times and feedback strengths. $P = 8 \times 10^{24}$ m^{-3}ns^{-1}.

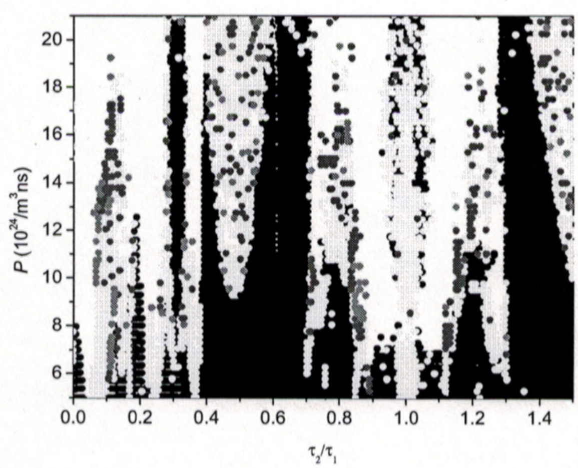

Fig. 8. State diagram of semiconductor laser with two external cavities in space of the ratios of the round trip times of the two external cavities and pump parameter. $\kappa_2/\kappa_1 = 1$.

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4. Conclusions

In conclusion, in this work we have studied complex dynamics in a semiconductor laser with external cavities. The second external cavity provides the additional possibility to control laser dynamics by varying its parameters. The analysis of bi-dimensional state diagrams of the laser with two external cavities have demonstrated that steady-state and different periodic orbits can be locked at some rational multiple of the phases of the two external cavities, forming Arnold tongues structure in the parameter spaces of the external cavity length, feedback strength and pump parameter. We believe that other structures inherent to the phase-locking phenomenon, such as Devil's staircases, Farey trees, etc., can be also found in a semiconductor laser with two external cavities. The results of this work can be of interest for laser engineering and some technological applications, for example, in communications for stabilization and control of laser operation.

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