# NEW OPTICAL SETUPS FOCUSED ON THE MEASUREMENT OF THE SHAPE AND DISPLACEMENT FIELDS 



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## Abstract of the Dissertation

# New optical setups focused on the measurement of the shape and displacement fields 

by

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This thesis presents the advances and achievements obtained during the PhD work. The focus of the work was to propose and create new implementations of well-known 3D shape recovery techniques such as Stereo Vision (SV), Digital Image Correlation (DIC) and Fringe Projection (FP) along with their algorithms. Since all these techniques are based on the capture of an image by a camera, a calibration process is needed in order to create a relationship between the image coordinates and the real world coordinates. In this thesis, the two utilized calibration methods are presented, Direct Linear Transformation (DLT) and Tsai's Method. Once the system is calibrated, it is possible to guarantee that the performed measurements correspond to a real coordinate system.

The main advantage of the optical metrology techniques is that they are contact-free so there is no alteration of the sample due to the measurement device. From there several applications come to mind: measurement of displacement fields, deformation, spatial location and topography. The proposals of this thesis are aimed to improve and simplify these tasks using the techniques mentioned before.

DIC is utilized to measure in-plane deformations using one camera, while a Stereo-DIC system is able to obtain the 3D shape and displacement fields of a sample, but its main disadvantage is the need of twin cameras in order to obtain a good correlation. In order to solve that problem we propose two possible solutions. The first consist of the use of a biprism to create the two perspectives of a scene using a single camera, with this change, we create an emulated SV system with twin cameras, since we utilize de the same camera for both perspectives; the only inconvenient is the distortion induced by the biprism but this can be fixed by calibration, as it is presented in this thesis. The second improvement is the use of FP with DIC because it has great sensibility to out-ofplane displacement, so when using both techniques together is possible to calculate 3D displacements fields of a deformed sample. If the area of interest of a sample is not a full-field view, but instead is only a point, we introduce a tracking system based in the detection of centroids of reflective markers which are utilized as points of interest to be located in a real world coordinate system.

Finally, a goal for several years has been the digital reconstruction of samples using the FP techniques, while most of the approaches depend of a static sample, for dynamic events it is necessary to obtain the whole view in one shot, or a multi-camera system. To solve this problem is presented a Panoramic Fringe Projection (PFP) system which allow us to obtain an all-around reconstruction of an object by using a conical mirror. In its respective chapter are presented the technical considerations for its implementation.

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## Justification

One of the most important applications of the optical metrology is the full-field measurement of surfaces and 3D reconstruction. Techniques such as Digital Image Correlation (DIC) are ideal for the measurement of the in-plane displacement, while its implementation for an out-of-plane measurement requires a stereo adaptation. On the other hand, Fringe Projection (FP) has higher resolution calculating out-of-plane displacements and shape recovery, but it lacks of in-plane sensitivity.

In FP, the shape recovery process is applied to only one perspective of the sample, in order to create an all-around calculation, it is necessary to move either the vision system or the sample, being the second the more convenient.

The proposed idea is to create a system that will allow us to measure the in-plane and out-of-plane displacements via DIC and FP. Although there are previous work about the fusion of the techniques, our contribution is to calculate all the data from one perspective of the object obtaining the allaround information, without moving the system nor the sample.

## Objectives

To create a system capable of recover the shape of an object in $360^{\circ}$ without moving the object or the system.

To implement a setup capable of measuring the in-plane and out-of-plane deformations of an object all over its surface from one perspective.

To implement a simplified Stereo-DIC system which allows to calculate 3D displacement fields and also capable of calculating the shape of an object.

To obtain the 3D displacement vectors in a full-field calculation utilizing DIC and FP techniques.

## Preface

This thesis presents the contributions to the field of optical metrology proposing new setups and processing algorithms of well-known full-field shape recovery techniques such as: Stereo Vision, Digital Image Correlation and Fringe Projection. The proposals have the goal of improving the existing methodologies, either the speed of acquisition or the quality of the measurement.

The thesis is presented as follows:
Chapter 1 provides a general introduction of the topics of interest, along with a bibliography review of the state of art and related work.

Chapter 2 presents a brief review of the calibration methods utilized to carry out our experiments. In particular, DLT-11 and Tsai-Zhang method are used in this thesis.

Chapter 3 contains the theory of DIC to realize 2D measurements. Experimental results using DIC and FP are presented for the same target object.

Chapter 4 presents the first contributions made by the author of the thesis. It is divided in two parts: The first part presents the theory and experimental results obtained with SV using a single camera and rotation of a sample with high reflectance points that make sample tracking possible. The second part presents the theory of Stereo-DIC and the proposal of a SV system utilizing a biprism. The distortion introduced by the biprism is corrected in order to obtain reliable results. Experimental results and conclusions are presented.

Chapter 5 describes the main contribution of the author. It contains the theory and adaptations of the FP technique required to implement a Panoramic Fringe Projection system able to obtain the $360^{\circ}$ topography of an object. The projected fringes correspond to concentric circular fringes by using of a conical mirror. The phase shifting method is used for phase recovery of the fringes. Experimental results and conclusions are included.

Chapter 6 summarizes the contributions of this research and proposes future works.

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## 1. General introduction

In optical metrology, one of the most important tools is the image acquisition of an object and one of the alternatives is the use of a camera. The capture of digital images makes possible applications such as strain calculation, pattern recognition, surface quality inspection, object classification and robotic navigation.

Nowadays, shape recovery research has been focused on improving the processing algorithms and simplifying the acquisition systems; in order to obtain more information with lesser hardware and shorter time. One of the applications of particular interest is the object 3D reconstruction, where the shape of a sample can be digitally obtained and utilized for different applications. With this goal in mind, many techniques have been developed by different research groups along the years, and the advances have been faster in the last years.

In this thesis, we reported the implementation of techniques for 3D digital reconstruction. The approach of these novel setups is to improve the data acquisition process either the speed or the simplification of the hardware, or both. The techniques under study are: Digital Image Correlation (DIC), Stereo Vision (SV) and Fringe Projection (FP). These techniques are well-known and present advantages as non-invasive, whole field, results in real time, high resolution, etc.

The project originally was the development of an optical system capable of recovering the shape of an object and measuring the displacement fields in and out of plane. To do so, the proposal was the mix of techniques such as DIC and FP, but since there are a lot of work already done in this field, the contribution relied on making the measurement in the whole surface of the object.

From there, several proposals were made, where they represented an improvement or an alternative of the existing techniques.

The first proposal to create our $360^{\circ}$ system was the use of SV to locate high reflective markers in order to create virtual reference planes, and then recover the topography all-around the object with the union of the different pieces of topography. The beginning of this project is presented in section 4.2.

Some issues were noted while implementing the SV system, such as it doesn't give a time resolved measurement, the need of cameras with very similar characteristics, etc. With it in mind, our work was focused then in trying to implement a SV system that utilized virtually twin cameras and time resolved measurements utilizing a biprism of an object which allowed us to have two perspectives of the same scene in one shot and calculate shape and fields of displacement using a SV-DIC system. This work is explained in section 4.3.

At the end the measurements created by the biprism needed the sample to rotate in order to make the proper $360^{\circ}$ analysis, but again, it wasn't time resolved. So another proposal was made, the creation of a FP system which permit us to measure the topography and deformations of an object all over its surface utilizing one perspective of the object. The solution was the use of a conical mirror with the sample in the center of it and projecting fringes on it. In that case, several matters
where found such as the sensitivity of the system and the coordinate transformation needed to obtain the correct reconstruction of the object. This matters are exposed in section 5.4

Finally, we created a DIC-FP system, capable of obtaining the displacement maps of a stressed sample by using DIC to calculate the in-plane deformations, and FP to calculate the out-of-plane deformations. This topic is presented in section 3.6

### 1.1 State of art

In order to use any camera-based technique, it is necessary to calibrate the optical system used in the capture of the images of the object in order to obtain the 3D information of the scene. The calibration obtains the relation between the captured image and the real measurements of the object. Along the years several proposals have been realized, one of them was the "Direct Linear Transformation" (DLT) proposed by Abdel-Aziz [1,2] in 1971, which established a transformation from a 3D coordinates called world coordinate system to a 2D coordinate system created by the projection of the scene in the image plane of the capturing device (in this case a camera), the method created the fundamentals of camera calibration and the definition of the intrinsic and extrinsic parameters. Then, in 1987 Tsai [3,4] created his calibration algorithm, based on the DLT and proposed a more versatile method which also included calibration for radial lens distortion.

For all purposes of calibration a calibration target is needed in most of the cases, which is an object of a known geometry that is used to calculate the parameters of the camera and the scene, that's why in 1997 Heikkilä et al [5,6] pointed that one of the most important steps in calibration, was the recognition of the points of interest in an object. They proposed the use of a circular printed pattern over a 3D calibration target and by centroid's detection, the calibration parameters were obtained. This method was also based on DLT.

In 1999 Zhang [7-9] created his calibration algorithm which was a more flexible technique since it only needed to observe a planar pattern (a checkerboard) at different orientations (at least two) and the motion of the camera or the sample need not be known. It also calculated the radial lens distortion.

From there, some other calibration methods have been developed depending on the application. Muñoz [50-52] performed a calibration without a reference target by using Bezier networks. In a particular case of a FP system, Liu et al. [10] utilized phase shifting to obtain the intrinsic and extrinsic parameters of the camera and projector as a stereo vision system. Legarda et al. [11] utilized structured light to also calibrate a camera-projector system by defining a unique coordinate system, which simplified the transformation to real coordinates. Zhang [12] proposed a method where it considers the projector as an inverse camera, Tavares [13] utilized projected fringes on a plane while it is displaced in the optical axis direction in order to obtain different perspectives and Falcao [14] created an software to calibrate a projector utilizing the calibration software developed by Bouget [9] (which uses Zhang's method). Several comparisons of the calibration methods have been done, such as the one made by Salvi [15] or the one from Remondino [16], but in general, it depends mostly of the application, the type of calibration that is more suitable to the setup.

Once the calibration is done, it depends on the application which technique is suitable for use. In the case of strain analysis and calculation of in-plane deformation, the technique of DIC has become very popular in the field of mechanics as mentioned by Bruck et al [17]. Sutton et al. [18] described DIC as a non-contacting method that require images of an object, stored in a digital form to perform an image analysis to extract full field shape, deformation and/or motion measurements. The popularity in the field of mechanics is that DIC works as an optical finite element method because is capable of calculating the gradients of deformation of any sample [19]. A more detailed description of the technique will be done in the next chapters, but it can be mentioned its importance in the mechanics field in applications such as: Identification of elastic properties made by Hild and Roux [20], "finite-element" displacement fields calculation by Besnard et al [21], and if the system has enough resolution, it is possible to apply the technique in a micro and nano scales as demonstrated by Pan et al [22].

It was found that Stereo Vision is very similar to DIC in obtaining 3D information, since the algorithms used in DIC are the same as in SV as was shown by Cardenas-Garcia [25]; nevertheless, the study of SV focuses on the arrangement and its components (hardware) while the data processing is done by DIC. Typical SV setups use two twin cameras which capture different perspectives of the same scene in order to obtain the topography through correlation [25-28]. The cameras can have their axes parallel or convergent and they can be epipolar or not, but two conditions remain, they must to have the same characteristics (contrast, brightness, resolution, etc.) and their fields of view overlap, which permits the correlation between the images obtained by each one of cameras.

While DIC presents a really good measurement quality when applied to the in-plane deformations, its capability to calculate 3D deformations and shape requires a SV system implementation, but its resolution is not good because of the kind of algorithm utilized to calculate the correlation. For out-of-plane deformation and shape recovery, FP presents a best result.

FP is a well-known high-resolution, non-contact technique used to retrieve the 3D topography of an object [33-37]. It is principle is the calculation of the deformation of synthetically generated and projected nearly ideal fringes, which deformation is directly proportional to the object's shape. The principal objective of the technique then remains in obtaining the phase maps, either by Fourier methods [38] or phase shifting algorithms [39]. By the nature of these methods, a process of phase unwrapping is needed, where a whole research area is dedicated for unwrapping procedure development. Depending on the quality of the unwrapped phase maps, the unwrapping process can be in 1D [33,40], 2D [33,41] or if the phase map is complex, it can use a predictor to detect its isophase [42], and these are only a few considerations.

As mentioned before, the FP is based on the projection of synthetically created fringe patterns using a multimedia projector, then, an image of the object is captured with the projected fringes on it. The pattern presents deformation due to the surface of the sample. To obtain the information of the topography, a phase calculation process has to be done: phase retrieval, phase unwrapping and conversion of the phase to real measurement units. All these considerations lead to some areas of interest about the technique. Just to mention a few of them, Zhang [36] proposed a real time system which uses phase shifting to obtain the phase by taking advantage of color fringes and 2 -step algorithms, Martinez et al [43] established an iterative algorithm for the correction of the variation
of the period, which introduces error in the phase calculation process, when the fringes are projected on big dimension objects, finally, Servin et al [44] proposed a low-noise $360^{\circ}$ reconstruction system where the sample is rotated and the different perspectives form a single phase map for each rotation step. In spite of the variety of applications, the main problem of fringe projection is that the technique is not useful to calculate in-plane deformations, nevertheless the fusion of FP and DIC to obtain full deformed information has already been suggested [45-47].

### 1.2 Related work

Since the equality of the cameras is so important when doing the correlation, one solution results obvious: the use of a single camera to make the stereo vision. Some proposals such as the capture of two perspectives by rotating the sample are useful if the object of interest is static [29], but most of the phenomena of interest are dynamic events. To solve this problem an interesting alternative is the implementation an arrangement of mirrors mounted in the camera, so the same camera captures the two perspectives, as suggested by Wang [30]. Lee et al [31] proposed a stereo system based on a biprism placed in front of a camera so that camera can capture the needed views. The only inconvenience in this work is lack of consideration of the significant image distortion introduced by the biprism. Also the SV is not limited to just two images, but as suggested by Xiao [32] a pyramidal prism also can be used to obtain multiple views of the object of even more complex geometries, but the use of different prisms reduce drastically the resolution of the measurement.

The applications of the algorithms of DIC are not limited to the 2D deformations, because by using a stereo vision system, it is possible to calculate also fields of out-of-plane deformation. As it is proposed by Genovese and et al. [23,24] the technique is even capable of recover $360^{\circ}$ information of a sample while it suffers a radial deformation. In their work, they used a biological sample (quasi-cylindrical) to calculate the fields of radial deformation in order to characterize its mechanical properties while the sample is stressed mechanically. Another technique applicable to the $360^{\circ}$ analysis is the rotation of the sample as the one proposed by Muñoz [53] by using a light line projection for 3D detection and Hu moments to avoid measurement errors.

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## 2. Calibration techniques

### 2.1 Introduction

Camera calibration is a necessary step in 3D computer vision in order to create a metric relation between the captured 2D image and the 3D scenario. This process consists of determination of the internal camera geometry and optical system characteristics, known as the intrinsic parameters, and the relative positions or/and orientation of the camera with respect to a world coordinate system, known as the extrinsic parameters [1-11].

The intrinsic parameters are completely dependent on the construction of the camera and the optics mounted on it. These intrinsic parameters are: the principal point on the sensor, focal distance of the optical system and the scale factors of the pixels with respect to a metric system. Depending on the used algorithm, the information about distortions caused by the lenses of the camera also can be included [1-9].

On the other hand, the extrinsic parameters are calculated based on the geometry of the arrangement. These parameters are: the rotation matrix and the translation vector. Both are utilized to transform a 3D real coordinate system into 2D coordinates in the sensor (also called image) plane. The extrinsic parameters may vary with the components of the arrangement, the geometry of the object with respect to the camera and the used algorithm. Much work has been done about the calibration and different ways to obtain the intrinsic and extrinsic parameters [10,11]. These methods can be roughly classified into two categories:

Photogrammetric calibration: These techniques are performed by observing a calibration target which geometry in 3D space is known a priori. It is mandatory to have a very good quality and well characterized calibration target in order to obtain accurate calculations, resulting this requirement as the main problem of these techniques.

Self-calibration: The techniques in this category do not use any calibration object. They are based on the movement of the camera around a static scene. In this case it is needed to have a well-known displacement of the camera, which represents the biggest inconvenient.

Both methods are based only on image processing [12-16]. Next sections present techniques used to calibrate the systems described in this thesis.

### 2.2 Direct linear transformation

The Direct Linear Transformation (DLT) is the basis of most of the reported calibration techniques. Its principle is to calculate the parameters from a known object (calibration target) [1,2]. The principle of the method is the pin-hole assumption shown in figure 2.1 where a camera can be characterized as a sensor and the optical system as a pin-hole where the rays pass through. The optical axis is considered perpendicular to the sensor until the pin-hole.


Figure 2.1 Pin-hole assumption of a camera.
The image acquisition process can be characterized very easily with this assumption as it can be seen in figure 2.2; let's consider a point $P$ located at $(x, y, z)$ of the world coordinates where its projection is captured by the sensor at the point $I_{l}(u, v)$ of the sensor coordinates. $C_{l}$ corresponds to the principal point, where the optical axis is traced perpendicular to the sensor plane and passes through the pin-hole $H_{l}$. The distance $f$ is the focal length of the optical system.


Figure 2.2 Point of interest's projection on the sensor plane.
With the ray trace is possible to see two collinear vectors:

$$
\begin{align*}
& \vec{A}=\overrightarrow{P H_{1}}  \tag{2.1}\\
& \vec{B}=\overrightarrow{H_{1} I_{1}} \tag{2.2}
\end{align*}
$$

and by applying the collinearity hypothesis

$$
\begin{equation*}
\vec{B}=c \vec{A} \tag{2.3}
\end{equation*}
$$

where $c$ is a scaling scalar. Both vectors can be express as:

$$
\vec{B}=\overrightarrow{H_{1} I_{1}}=\left(\begin{array}{c}
u-u_{0}  \tag{2.4}\\
v-v_{0} \\
f
\end{array}\right)
$$

in the sensor coordinate system. On the other hand

$$
\vec{A}=\overrightarrow{P H_{1}}=\left(\begin{array}{l}
x  \tag{2.5}\\
y \\
z
\end{array}\right)
$$

in the world coordinate system. To make the transformation from world coordinates to sensor coordinates we apply a rigid body transformation where the vector of interest is multiplied by a rotation matrix and then added a translation vector. Applying this transformation to equation (2.5) we obtain

$$
\vec{A}=\overrightarrow{P H_{1}}=\left(\begin{array}{lll}
R_{11} & R_{12} & R_{13}  \tag{2.6}\\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33}
\end{array}\right)\left(\begin{array}{l}
x \\
y \\
z
\end{array}\right)+\left(\begin{array}{l}
t_{x} \\
t_{y} \\
t_{z}
\end{array}\right)
$$

By substituting (2.4) and (2.6) in (2.3) we obtain the relationship between both vectors.

$$
\left(\begin{array}{c}
u-u_{0}  \tag{2.7}\\
v-v_{0} \\
f
\end{array}\right)=c\left[\left(\begin{array}{lll}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33}
\end{array}\right)\left(\begin{array}{l}
x \\
y \\
z
\end{array}\right)+\left(\begin{array}{l}
t_{x} \\
t_{y} \\
t_{z}
\end{array}\right)\right]
$$

All the parameters expressed in (2.6) have to be expressed in the same units, in this case metric. Since $u$ and $v$ are both coordinates of the sensor they have to be transformed into metric system by the use of scaling factors $\lambda_{x}^{-1}$ and $\lambda_{y}^{-1}$ which are the size of a pixel in the sensor. Rearranging (2.6) and adding the scale factors of the pixels we obtain:

$$
\begin{gather*}
\left(u-u_{0}\right)=c \lambda_{x}\left(R_{11} x+R_{12} y+R_{13} z+t_{x}\right)  \tag{2.8}\\
\left(v-v_{0}\right)=c \lambda_{y}\left(R_{21} x+R_{22} y+R_{23} z+t_{y}\right)  \tag{2.9}\\
f=c\left(R_{31} x+R_{32} y+R_{33} z+t_{z}\right) \tag{2.10}
\end{gather*}
$$

where $c$ can be expressed

$$
\begin{equation*}
c=\frac{f}{\left(R_{31} x+R_{32} y+R_{33} z+t_{z}\right)} \tag{2.11}
\end{equation*}
$$

substituting (2.11) in (2.8) and (2.9) we can rewrite the expressions into

$$
\begin{align*}
& \left(u-u_{0}\right)=f \lambda_{x} \frac{\left(R_{11} x+R_{12} y+R_{13} z+t_{x}\right)}{\left(R_{31} x+R_{32} y+R_{33} z+t_{z}\right)}  \tag{2.12}\\
& \left(v-v_{0}\right)=c f \frac{\left(R_{21} x+R_{22} y+R_{23} z+t_{y}\right)}{\left(R_{31} x+R_{32} y+R_{33} z+t_{z}\right)} \tag{2.13}
\end{align*}
$$

These final expressions give a general idea of what calibration must do. In order to obtain the real measurements of an object, first we have to find the extrinsic and intrinsic parameters of the camera, which in this case are $\lambda_{x}, \lambda_{y}, u_{0}, v_{0}, f, \boldsymbol{R}, t_{x}, t_{y}$ and $t_{z}$.

In the DLT method the goal is to simplify the solution to both equations (2.12) and (2.13) by substituting the unknown parameters with a series of constants called $L_{n}$. The basic method uses 11 parameters, and depending how many corrections we wish to do to the optical system (such as spherical distortion, astigmatism, etc) the number can be increased. For our case we will only focus on 11 .

Equations (2.12) and (2.13) can be rewritten as

$$
\begin{align*}
& u=\frac{L_{1} x+L_{2} y+L_{3} z+L_{4}}{L_{9} x+L_{10} y+L_{11} z+1}  \tag{2.14}\\
& v=\frac{L_{5} x+L_{6} y+L_{7} z+L_{8}}{L_{9} x+L_{10} y+L_{11} z+1} \tag{2.15}
\end{align*}
$$

where

$$
\begin{array}{ll}
L_{1}=\frac{u_{0} R_{31}+f \lambda_{x} R_{11}}{t_{z}} & L_{2}=\frac{u_{0} R_{32}+f \lambda_{x} R_{12}}{t_{z}} \quad L_{3}=\frac{u_{0} R_{33}+f \lambda_{x} R_{13}}{t_{z}}  \tag{2.16}\\
L_{4}=\frac{u_{0} t_{z}+f \lambda_{x} t_{x}}{t_{z}} & L_{5}=\frac{v_{0} R_{31}+f \lambda_{y} R_{21}}{t_{z}} \quad L_{6}=\frac{u_{0} R_{32}+f \lambda_{y} R_{22}}{t_{z}} \\
L_{7}=\frac{u_{0} R_{33}+f \lambda_{y} R_{23}}{t_{z}} & L_{8}=\frac{v_{0} t_{z}+f \lambda_{y} t_{y}}{t_{z}} \quad L_{9}=\frac{R_{31}}{t_{z}} L_{10}=\frac{R_{32}}{t_{z}} L_{11}=\frac{R_{33}}{t_{z}}
\end{array}
$$

To calculate the 11 parameters it is necessary to express (2.14) and (2.15) in matrix way

$$
\left[\begin{array}{ccccccccccc}
x & y & z & 1 & 0 & 0 & 0 & 0 & -x u & -y u & -z u  \tag{2.17}\\
0 & 0 & 0 & 0 & x & y & z & 1 & -x v & -y v & -z v
\end{array}\right]\left[\begin{array}{c}
L_{1} \\
L_{2} \\
\vdots \\
L_{11}
\end{array}\right]=\left[\begin{array}{c}
u \\
v
\end{array}\right]
$$

In (2.17) is possible to notice that it is necessary to know a priori the dimensions of an object in order to calculate the $L$ parameters. That's the importance of the calibration target, since we already know the coordinates of each of its points $(x, y, z)$ and their projection in the image plane $(u, v)$ we can solve the system. Each point generates 2 equations and a minimum of six points is required to find the 11 parameters.

Once the calibration has been carried out, it is possible to make the reconstruction of the object obtaining the world coordinates of the object by rearranging equations (2.14) and (2.15)

$$
\left[\begin{array}{lll}
\left(u L_{9}-L_{1}\right) & \left(u L_{10}-L_{2}\right) & \left(u L_{11}-L_{3}\right)  \tag{2.18}\\
\left(v L_{9}-L_{5}\right) & \left(v L_{10}-L_{6}\right) & \left(v L_{11}-L_{7}\right)
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
L_{4}-u \\
L_{8}-v
\end{array}\right]
$$

For each point captured by the camera we have two equation to obtain the 3 unknowns $(x, y, z)$. In order to solve the system it is required a minimum of 2 cameras to find the unknowns, to do so, the same calibration process is done to the second camera and both are put together so solve the system.

In figure 2.3 the different perspectives of the object are shown, which is the principle of the stereovision, where the subindexes $R$ and $L$ correspond to the right and left cameras respectively. As mentioned in [6], the detection of the points of interest for the calibration are detect via an image processing. The captured images are binarized allowing the detection of their centroids. The coordinates of each centroid are associated to the variables $u_{R}, v_{R}, u_{L}$ and $v_{L}$.


Figure 2.3 Reconstruction process of an object utilizing stereovision.
The equations to obtain the world coordinates are expressed as a matrix:

$$
\left[\begin{array}{ccc}
\left(u_{L} L_{L 9}-L_{L 1}\right) & \left(u_{L} L_{L 10}-L_{L 2}\right) & \left(u_{L} L_{L 11}-L_{L 3}\right)  \tag{2.19}\\
\left(v_{L} L_{L 9}-L_{L 5}\right) & \left(v_{L} L_{L 10}-L_{L 6}\right) & \left(v_{L} L_{L 11}-L_{L 7}\right) \\
\left(u_{R} L_{R 9}-L_{R 1}\right) & \left(u_{R} L_{R 10}-L_{R 2}\right) & \left(u_{R} L_{R 11}-L_{R 3}\right) \\
\left(v_{R} L_{R 9}-L_{R 5}\right) & \left(u_{R} L_{R 10}-L_{R 2}\right) & \left(v_{R} L_{R 11}-L_{R 7}\right)
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
L_{L 4}-u_{L} \\
L_{L 8}-v_{L} \\
L_{R 4}-u_{R} \\
L_{R 8}-v_{R}
\end{array}\right]
$$

where $L_{n}$ are known from calibration, $u$ and $v$ are known from the captured images.
By solving 2.19 is possible to obtain the spatial location of each point in a $X, Y, Z$ Cartesian space. To solve the vision parameters individually, expressions 2.16 are needed to be substituted with the known $L_{n}$ parameters.

As mentioned before the 11 parameters correspond to a standard DLT calibration, while it has been reported 12 to 14 parameters to include calibration of 3rd, 5th and 7th order optical distortions, while 15 and 17 parameters to fix de-centering distortions [1,2].

### 2.3 Tsai's calibration method

Tsai's method was first proposed in 1987 [3] as an improvement to the existing methods at that time. Next it was improved by Heikkilä in 1997 [5] and Zhang in 1999 [7], and since then the
method hasn't seen any major changes $[6,8,9]$. In this section we will focus on explanation of the original method.

Tsai's method for camera calibration recovers the interior orientation, the exterior orientation, the power series coefficients for distortion, and an image scale factor that best fits the measured image coordinates corresponding to known target point coordinates. This is done in stages, starting off with closed form least-squares estimates of some parameters and ending with an iterative non-linear optimization of all parameters simultaneously using these estimates as starting values. Importantly, the error in the image plane that is minimized.

Details of the method are different for planar targets than for targets occupying some volume in space. Accurate planar targets are easier to make, but lead to some limitations in camera calibration.

The intrinsic parameters, also known as interior orientation, the intrinsic parameters create the relationship between camera-centric coordinates and image coordinates. The camera has its origin at the center of the sensor, at least theoretically, its $z$ axis along the optical axis, and its $x$ and $y$ axes parallel to the $x$ and $y$ axes of the image.

Camera coordinates and image coordinates are related by the perspective projection equations:

$$
\begin{align*}
& \frac{x_{I}-x_{0}}{f}=\frac{x_{C}}{z_{C}}  \tag{2.20}\\
& \frac{y_{I}-y_{0}}{f}=\frac{y_{C}}{z_{C}} \tag{2.21}
\end{align*}
$$

where $f$ is the principle distance (distance from the center of projection to the image plane), and ( $x_{0}$, $y_{0}$ ) is the principle point (foot of the perpendicular from the center of projection to the image plane). That is, the center of projection is at $\left(x_{0}, y_{0}, f\right)^{\mathrm{T}}$, as measured in the image coordinate system.

Interior orientation has three degrees of freedom. The problem of interior orientation is the recovery of $x_{0}, y_{0}$, and $f$. This is the basic task of camera calibration. However, in practice we also need to recover the position and altitude of the calibration target in the camera coordinate system.

The extrinsic parameters, also known as the exterior orientation, create the relationship between a scene-centered coordinate system and a camera-centered coordinate system. The transformation from scene to camera consists of a rotation and a translation. This transformation has six degrees of freedom, three for rotation and three for translation.

The scene coordinate system can be any system convenient for the particular design of the target. In the case of a planar target, the $z$ axis is chosen perpendicular to the plane, and $z=0$ in the target plane.

If $\boldsymbol{r}_{S}$ are the coordinates of a point measured in the scene coordinate system and $\boldsymbol{r}_{C}$ coordinates measured in the camera coordinate system, then

$$
\begin{equation*}
\boldsymbol{r}_{C}=\boldsymbol{R}\left(\boldsymbol{r}_{S}\right)+\boldsymbol{t} \tag{2.22}
\end{equation*}
$$

where $\boldsymbol{t}$ is the translation vector and $\boldsymbol{R}$ the rotation. Expressing equation (2.22) in matrix form:

$$
\left(\begin{array}{l}
x_{C}  \tag{2.23}\\
y_{C} \\
z_{C}
\end{array}\right)=\left(\begin{array}{lll}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33}
\end{array}\right)\left(\begin{array}{l}
x_{S} \\
y_{S} \\
z_{S}
\end{array}\right)+\left(\begin{array}{c}
t_{x} \\
t_{y} \\
t_{z}
\end{array}\right)
$$

where $\boldsymbol{r}_{C}=\left(x_{C}, y_{C}, z_{C}\right)^{T}, \boldsymbol{r}_{S}=\left(x_{S}, y_{S}, z_{S}\right)^{T}$ and $\boldsymbol{t}=\left(t_{x}, t_{y}, t_{z}\right)^{T}$.
The unknowns to be recovered are the translation vector $\boldsymbol{t}$ and the rotation matrix $\boldsymbol{R}$.
If we combine the equations for intrinsic and extrinsic parameters we obtain:

$$
\begin{align*}
& \frac{\left(x_{I}-x_{0}\right)}{f}=\frac{\left(R_{11} x_{S}+R_{12} y_{S}+R_{13} z_{S}+t_{x}\right)}{\left(R_{31} x_{S}+R_{32} y_{S}+R_{33} z_{S}+t_{z}\right)}  \tag{2.24}\\
& \frac{\left(x_{I}-v_{0}\right)}{f}=\frac{\left(R_{21} x_{S}+R_{22} y_{S}+R_{23} z_{S}+t_{y}\right)}{\left(R_{31} x_{S}+R_{32} y_{S}+R_{33} z_{S}+t_{z}\right)} \tag{2.25}
\end{align*}
$$

### 2.3.1 Distortion

Projection in an ideal imaging system is governed by the pin-hole model. Real optical systems suffer from a number of inevitable geometric distortions. In optical systems made of spherical surfaces, with centers along the optical axis, a geometric distortion occurs in the radial direction. A point is imaged at a distance from the principle point that is larger (pin-cushion distortion) or smaller (barrel distortion) than predicted by the perspective projection equations; the displacement increasing with distance from the center. It is small for directions that are near parallel to the optical axis, growing as some power series of the angle. The distortion tends to be more noticeable with wide-angle lenses than with telephoto lenses.

The displacement due to radial distortion can be modeled using the equations:

$$
\begin{align*}
& \delta_{x}=x\left(\kappa_{1} r^{2}+\kappa_{2} r^{4}+\cdots\right)  \tag{2.26}\\
& \delta_{y}=y\left(\kappa_{1} r^{2}+\kappa_{2} r^{4}+\cdots\right) \tag{2.27}
\end{align*}
$$

where $x$ and $y$ are measured from the center of distortion, which is assumed to be the principal point.

Electro-optical system typically also suffers tangential distortion and it grows with distance from the center of distortion.

$$
\begin{gather*}
\delta_{x}=-y\left(\epsilon_{1} r^{2}+\epsilon_{2} r^{4}+\cdots\right)  \tag{2.28}\\
\delta_{y}=x\left(\epsilon_{1} r^{2}+\epsilon_{2} r^{4}+\cdots\right) \tag{2.29}
\end{gather*}
$$

In calibration, the goal is to recover coefficients $\kappa_{n}$ and $\varepsilon_{n}$ of these power series.

### 2.3.2 Calibration process

In calibration, a target of known geometry is imaged. Correspondences between target points and their images are obtained. This forms the basic data on which the calibration is based.

Tsai's method first tries to obtain estimates of as many parameters as possible using linear leastsquares fitting methods. This is convenient and fast since such problems can be solved using the pseudo-inverse matrix.

In this initial step, constraints between parameters (such as the orthonormality of a rotation matrix) are not enforced, and what is minimized is not the error in the image plane, but a quantity that simplifies the analysis and leads to linear equations. This does not affect the final result, however, since these estimated parameter values are used only as starting values for the final optimization.

In a subsequent step, the rest of the parameters are obtained using a nonlinear optimization method that finds the best fit between the observed image points and those predicted from the target model. Parameters estimated in the first step are refined in the process.

Details of the calibration method are different when the target is planar then when it is not. Accurate planar targets are easier to make and maintain than three-dimensional targets, but limit the calibration in ways that will become apparent.

Initially we assume that we have a reasonable estimate of the position of the principal point ( $x_{0}, y_{0}$ ). This point is usually near the middle of the CCD or CMOS sensor. We refer coordinates to this point using

$$
\begin{align*}
& x_{I}^{\prime}=x_{I}-x_{0}  \tag{2.30}\\
& y_{I}^{\prime}=y_{I}-y_{0} \tag{2.31}
\end{align*}
$$

so that

$$
\begin{align*}
& \frac{x_{I}^{\prime}}{f}=\frac{x_{C}}{z_{C}}  \tag{2.31}\\
& \frac{y_{I}^{\prime}}{f}=\frac{y_{C}}{z_{C}} \tag{2.32}
\end{align*}
$$

When we consider only the direction of the point in the image as measured from the principle point. This yields a result that is independent of the unknown principle distance $f$. It is also independent of radial distortion.

$$
\begin{equation*}
\frac{x_{I}^{\prime}}{y_{I}^{\prime}}=\frac{x_{C}}{y_{C}} \tag{2.33}
\end{equation*}
$$

With a planar target it is possible to arrange the coordinate system such that $z_{S}=0$ for points on the target. This means the products $R_{13}, R_{23}$ and $R_{33}$ are out of the equations, then, considering the relation in (2.33) for the image coordinates we obtain:

$$
\begin{equation*}
\frac{x_{I}^{\prime}}{y_{I}^{\prime}}=\frac{\left(R_{11} x_{S}+R_{12} y_{S}+t_{x}\right)}{\left(R_{21} x_{S}+R_{22} y_{S}+t_{y}\right)} \tag{2.34}
\end{equation*}
$$

which after cross multiplying becomes:

$$
\begin{equation*}
\left(x_{S} y_{I}^{\prime}\right) R_{11}+\left(y_{S} y_{I}^{\prime}\right) R_{12}+y_{I}^{\prime} t_{x}-\left(x_{S} x_{I}^{\prime}\right) R_{21}+\left(y_{S} x_{I}^{\prime}\right) R_{22}+x_{I}^{\prime} t_{y}=0 \tag{2.35}
\end{equation*}
$$

a linear homogeneous equation with six unknowns: $R_{11}, R_{12}, R_{21}, R_{22}, t_{x}$ and $t_{y}$.
The coefficients in this equation are products of components of corresponding scene and image coordinates. We obtain one equation for every correspondence between a target point and an image point. These approximations now leave the task of estimating the rotation matrix based on its top left $2 \times 2$ sub-matrix.

Since the rotation matrix is supposed to be orthonormal, we have

$$
\begin{gather*}
R_{11}^{2}+R_{12}^{2}+R_{13}^{2}=k^{2}  \tag{2.36}\\
R_{21}^{2}+R_{22}^{2}+R_{23}^{2}=k^{2}  \tag{2.37}\\
R_{11} R_{21}+R_{12} R_{22}+R_{13} R_{23}=0 \tag{2.38}
\end{gather*}
$$

where

$$
\begin{align*}
& k^{2}=\frac{1}{2}\left[\left(R_{11}^{2}+R_{12}^{2}+R_{21}^{2}+R_{22}^{2}\right)\right.  \tag{2.39}\\
&\left.+\sqrt{\left(\left(R_{11}-R_{22}\right)^{2}+\left(R_{12}+R_{21}\right)^{2}\right)\left(\left(R_{11}+R_{22}\right)^{2}+\left(R_{12}-R_{21}\right)^{2}\right)}\right]
\end{align*}
$$

We normalize the first two rows of the rotation matrix by dividing by $k$. Finally, we make up the third row by taking the cross-product of the first two rows. To avoid sign ambiguities in the calculation of $R_{13}$ and $R_{23}$ we can get the sign of the product $R_{13} R_{23}$ using the relation

$$
\begin{equation*}
R_{13} R_{23}=-\left(R_{11} R_{22}+R_{12} R_{22}\right) \tag{2.40}
\end{equation*}
$$

Finally to calculate $t_{z}$ and $f$ we cross multiply equations (2.24) and (2.25), leaving us a 2 equation system which can be solved with one correspondence between target and image.

After obtaining all the parameters, a non-linear optimization can be applied to minimize the image errors, that is, the difference between the observed image positions and the positions predicted on the known target coordinates.

### 2.4 Dynamic calibration

The methods mentioned in sections 2.2 and 2.3 correspond to a static calibration, where the optical system is fixed. In applications such as the mobile ones, it is imperative to do a recalibration for each displacement of the system. The inconvenient for this situation is that on a mobile environment it is not possible to have a calibration target. Muñoz et al. [12-16] proposed an online self-
calibration technique where the vision parameters are determined during the vision task. Several applications such as laser scanning and mobile stereo vision need this kind of recalibration.

The principle of this method is based on the deduction of the image coordinates with the transformation presented in equations 2.6 and 2.22. This criterion is canceled when the camera is translated in any axis. Hence, the translation vector $\mathbf{t}$ is modified and it has to be recalibrated. Without a reference to recalibrate, the vision parameters are deduced with the use of neuronal networks which consider an image plane parallel to the reference plane. A more detailed description of the technique can be found in [12-16].

### 2.5 Conclusions

The calibration algorithms presented in this chapter correspond to the ones utilized during the elaboration of the main contributions of this research, which are exposed in Chapters 4 and 5. DLT was utilized for the calibration of the biprism and the PFP system, while a mix of Tsai's method and DLT was utilized in the DIC-FP implementation of section 3.6.

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## 3. Digital image correlation

### 3.1 Introduction

The Digital Image Correlation (DIC) is a non-contact technique utilized for full-field deformation measurement. It uses image processing to find the position of a certain point of interest in different images of the same scene in order to calculate the corresponding displacements they could present. In order to do the calculation, the presence of a natural or synthetic pattern on the sample surface is necessary [1-3].

The DIC process can be done in a 2D or 3D way, depending of the application. In 2D DIC a single camera is utilized to measure in-plane displacements of a surface by obtaining the information of two or more temporal images of the same object. In the case of 3D DIC, the use of at least two images of different perspectives of the 3D sample is necessary. In order to obtain these perspectives, several proposals have been made [2] as it will be presented in chapter 4. Also for DIC method some conditions have to be considered: a) the area of analysis must be in the field of view for both perspectives (in order to make the correlation), b) the images have to be in focus during deformation, c) the perspectives have to be taken with the same acquisition system.

DIC has a large number of practical applications, to name just a few, image matching is used to solve problems in industrial process control, biological growth phenomena, geological mapping, stereo vision, optical flow calculation, solid mechanics, velocimetry, fluid mechanics and even 3D reconstruction.

In this chapter is presented a brief description of the DIC technique, the algorithms utilized in this thesis, some considerations of the method and experimental results.

### 3.2 Image matching methods

It is relatively simple for a human observer to identify motion in successive images, but it is not straightforward to formulate the process in mathematical terms, and therefore, many different approaches exist. In this section we present the two widely used methods.

### 3.2.1 Differential method

To illustrate the problem of motion estimation, let's consider a one-dimensional problem as illustrated in figure 3.1 where $G(x, t)$ is the intensity distribution on the object as a function of time. If the motion is sufficiently small, it is possible to approximate the gray intensity values around a point of interest by a first order Taylor expansion

$$
\begin{equation*}
G(x+\Delta x, t)=G(x, t)+\frac{\delta G}{\delta x} \Delta x \tag{3.1}
\end{equation*}
$$

If the object moves with a constant velocity $u$, a gray value will be displaced by an amount $\Delta x=$ $u \Delta t$ in a time interval $\Delta t$, and the gray value distribution after a time step $\Delta t$ is merely a shifted copy of the original gray value distribution (considering an ideal variation if the intensity)

$$
\begin{align*}
\Delta G & =G(x, t+\Delta t)-G(x, t) \\
& =G(x-u \Delta t, t)-G(x, t) \\
& =G(x-\Delta x, t)-G(x, t) \tag{3.2}
\end{align*}
$$



Figure 3.1 Differential motion estimation of a 1D problem.
By substituting equation (3.1) into (3.2), the change in gray value $\Delta G$ can be expressed as a function of the slope of the intensity as follows:

$$
\begin{equation*}
\Delta G=-\frac{\delta G}{\delta x} \Delta x \tag{3.3}
\end{equation*}
$$

and thus obtain an estimate for the motion

$$
\begin{equation*}
\Delta x=-\frac{\Delta G}{\frac{\delta G}{\delta x}} \tag{3.4}
\end{equation*}
$$

For small motions, the displacement is given by the change in intensity divided by the slope of the intensity pattern. By substituting $\Delta x=u \Delta t$ into equation (3.4), we can obtain an estimate velocity

$$
\begin{equation*}
u \Delta t=-\frac{\Delta G}{\frac{\delta G}{\delta x}} \tag{3.5}
\end{equation*}
$$

and after taking the limit $\Delta t \rightarrow 0$ we obtain

$$
\begin{equation*}
\frac{\delta G}{\delta t}+u \frac{\delta G}{\delta x}=0 \tag{3.6}
\end{equation*}
$$

For the 2D velocity $\boldsymbol{v}$, the same derivation can be followed using the Taylor expansion $G(\boldsymbol{x}+$ $\Delta \boldsymbol{x})=G(\boldsymbol{x})+\Delta \boldsymbol{x} \cdot \nabla G$, obtaining the following equation

$$
\begin{equation*}
\frac{\delta G}{\delta t}+\boldsymbol{v} \cdot \nabla G=0 \tag{3.7}
\end{equation*}
$$

This equation is known as the brightness change constraint equation for the optical flow, and has been the center of a large body of research [2].

### 3.2.2 Template matching

The template matching method consists of making a motion estimation based on minimization of the gray value difference between a small subset from one image (template) and a displaced copy in another image (figure 3.2). We assume that between the two images no lighting changes occur. Let's denote the reference image from which the template is taken by $F$, and the image after displacement as $G$. Then, the template is correlated with a small portion of the deformed image, called analysis window, by using the Fast Normalized Cross-Correlation $[4,5]$

$$
\begin{equation*}
c(u, v)=\frac{\sum_{x, y}[a(x, y)-\bar{a}][t(x-u, y-v)-\bar{t}]}{\sqrt{\sum_{x, y}[a(x, y)-\bar{a}]^{2} \sum_{x, y}[t(x-u, y-v)-\bar{t}]^{2}}} \tag{3.8}
\end{equation*}
$$

where $a$ is the analysis window of $G, \bar{a}$ is the mean value of the analysis window, $t$ is the template window of $F$ and $\bar{t}$ is the mean of the template. Finally, $c(u, v)$ is the correlation coefficient located at $u$ and $v$ distance from $x$ and $y$ respectively. Where $c$ reaches its maximum, $u$ and $v$ become $d_{x}$ and $d_{y}$ which are the estimates of the average motion of the subset $t$. Based on eq. 3.8, the resulting matrix $c$ contains the correlation coefficients, which can range in value from -1.0 to 1.0.


Figure 3.2 Stress test applied to a dog-bone latex sample. a) Undeformed sample, b) deformed sample.

### 3.3 Subset shape functions

The image matching algorithms are limited to the determination of the average in-plane displacement of a typically square subset between two images. However in many applications the
measurement of complex displacement fields is of interest, and the specimen might experience elongation, compression, shear or rotation. In other words, an initially square reference subset might assume a considerably distorted shape in a later image after deformation. A subset shape function transform pixel coordinates in the reference subset into coordinates in the image after deformation.

### 3.3.1 Polynomial shape functions

A simple approach to account for increasingly complex subset deformations is to use shape functions that are polynomials in the subset coordinates. This family of subset functions includes pure displacements (zero-order polynomial) and affine transformations (first-order polynomial), but can be extended to allow for quadratic and higher-order polynomial functions. Derivatives of the polynomial shape functions with respect to its parameters are easily found as products of gray value derivatives and powers of subset coordinates. Another advantage of the use of this functions is, since the image correlation algorithms do not directly fit a displacement field to measured displacement data, its minimization of the error function defined by the intensity distribution.

The central assumption is that the true displacement field is best approximated by the local shape function when the error function defined in terms of the gray value attains its minimum. In this case, the subset used for image correlation defines a polynomial low-pass filter applied to the displacement field encoded in the images, which can also be defined by Bezier functions [18].

### 3.4 Image matching problems

### 3.4.1 Aperture problem

Generally it is difficult to find the correspondence of a single pixel of an image in a second image. Typically the gray value of a single pixel can be found at thousands of other pixels in the second image, and there is no unique correspondence. Therefore it is preferable to consider finding the correspondence of a small neighborhood around the pixel of interest. While the neighborhood provides additional information, the matching problem still may not be unique.



Figure 3.3 Aperture problem in image matching. a) A point in a line can be matched with any point on the displaced line, $b$ ) the enlarged aperture allows to observe the end-points of the line and determine the unique motion vector.

Figure 3.3a ilustrates the problem for a line in an image. Even though the component of the motion vector that is perpendicular to the line can be resolved, the motion along the line can't. This ambiguity is commonly referred as the aperture problem. In figure 3.3 b the aperture is increased revealing the end points of the line, so the motion vector of the line can be uniquely determined.

### 3.4.2 The correspondence problem

There are many situations where a unique correspondence between features in two images can't be established. Figure 3.4 shows two cases of correspondence problem. Figure 3.4 a shows a repeating periodic pattern of dots, in which case the motion calculation has as many solutions as pattern repetitions. A solution to this problem is the increase of the aperture size beyond periodic pattern, and the correspondence becomes unique and the motion vector can be resolved.

Figure 3.4b shows a textureless structure undergoing a deformation In this case it is not possible to obtain any motion information inside its boundaries since no features are present. But due to the deformation, it is even impossible to determine motion vectors on the boundary of the structure.


Figure 3.4 Correspondence problem to a) repeating periodic structure and b) textureless deforming structure.

### 3.4.3 Speckle pattern

To solve the correspondence problem uniquely, the object surface has to exhibit certain properties. The ideal surface texture should therefore be isotropic, i.e., it should not have a preferred orientation. Furthermore, it has been shown that repeating textures can lead to misidentification problems. The preferred surface texture should therefore be non-periodic. These requirements naturally lead to the use of random textures, such as the speckle pattern formed when a coherently illuminated surface is viewed through an aperture. The patterns commonly applied typically resemble laser speckle patterns to some degree. However, the patterns used in digital image correlation adhere to the surface and deform with the surface, and therefore no loss of correlation occurs even under large translations and deformations. Some examples of speckle patterns are shown in figure 3.2. One of the key features of good speckle patterns is their high information content. Since the entire surface is textured, information for pattern matching is available everywhere on the surface, and not only on a comparatively sparse grid. This permits the use of a relatively small aperture for pattern matching, commonly referred to as a subset or window.

### 3.5 Stereo-DIC

Stereo Digital Image Correlation (Stereo-DIC) is a well-assessed non-contact optical technique capable of performing shape and 3D deformation measurements on length scales ranging from microns to meters with a time resolution up to nanoseconds [2,6-13]. A series of image pairs captured from two different views of the object are used to locate and track a given set of surface points during motion and/or deformation. To correlate the two stereo-views, DIC requires the object surface to be provided with a random pattern of dark and light features. This allows finding the best match between corresponding points in the two images by comparing the local grey scale distribution of square pixel subsets on the basis of the normalized cross correlation coefficient [2]. An efficient matching operation requires the two images to be similar in terms of speckle pattern appearance. This can be achieved by using a pair of 'twin' cameras (with identical settings) and proper illumination, or by capturing both views simultaneously with one single camera and additional external optical devices. The latter option strongly facilitates correspondence-searching reliability and further eliminates the need for hardware and software synchronization entailed in time-resolved measurements. This topic will be discussed more in detail in section 4.2.

### 3.6 DIC and fringe projection

In order to demonstrate the scope of the DIC technique, a contribution is presented in this section. As mentioned before, DIC is capable of recovering out-of-plane deformation and shape, but its main limitation is the sub-sampled scan by performing the template matching algorithms. To cover this limitation. The mix of the fringe projection technique and DIC is proposed.

### 3.6.1 Introduction

The following work was done in order to analyze a variety of elastic samples for their in-plane and out-of-plane displacement fields. The calculus was done by using DIC to obtain the in-plane deformation fields while a fringe projection (FP) technique (which will be described in section 5.1) was utilized to calculate the out-of-plane deformation [15-17]. The object under analysis was a latex sample which was exposed to a known spherical deformation in the z -axis.


Figure 3.5 DIC-FP arrangement.

### 3.6.2 Methodology

Figure 3.5 shows the utilized arrangement, where a CCD camera and a multimedia projector were used. $p$ denotes the period of the fringes, $\theta$ is the angle between the camera and the projector and $\Delta z$ refers to the out-of-plane displacement of the sample, which is the difference between the reference state and the deformed state.

The projector is located below the camera, to obtain sensitivity in the system, the projected fringes were horizontal as it can be seen in figure 3.6a. In order to utilize the DIC algorithm, the sample shown in figure 3.7 was previously prepared with a printed (sprayed) speckle pattern to allow the correlation, following the considerations mentioned in 3.4.3. The system was calibrated using the Bouget [21] and Falcao [22] calibration toolboxes.

### 3.6.2.1 Out-of-plane displacement

The out-of-plane deformation was calculated by using a fringe pattern projected on the sample (figure 3.6) and a 4 -step algorithm was utilized to obtain the phase and associate the deformation to it. Since the interest is the calculation of the deformation of the sample from a reference state to a deformed state, two sets of images were taken, one for the reference ( $I_{R 1}, I_{R 2}, I_{R 3}, I_{R 4}$ ), and one for the deformed state ( $I_{D 1}, I_{D 2}, I_{D 3}, I_{D 4}$ ). Both sets were processed in the same way.

Since the images captured by the camera are represented by the expression [19]:

$$
\begin{equation*}
I_{N}(x, y)=a(x, y)+b(x, y) \cos \left[\phi(x, y)+\omega_{0} n\right] \forall n \in[1, N] \tag{3.9}
\end{equation*}
$$

Where $a(x, y)$ is the background, $b(x, y)$ the contrast, $\phi(x, y)$ the phase of the object, $N=4$ and $\omega_{0}=\pi / 2$. Then, the phase was retrieved by using 4 -step algorithm [19]:

$$
\begin{equation*}
\phi(x, y)=\tan ^{-1}\left[\frac{I_{1}(x, y)-I_{3}(x, y)}{I_{2}(x, y)-I_{4}(x, y)}\right] \tag{3.10}
\end{equation*}
$$

The resulting phase map is directly proportional to the topography of the object by means of [20]:

$$
\begin{equation*}
z(x, y)=\frac{\phi(x, y)}{2 \pi} \cdot \frac{p}{\tan \theta} \tag{3.11}
\end{equation*}
$$

For each state of the sample, a different topography is associated as $z_{R}$ for the reference and $z_{D}$ for the deformed state. The calculated phase contains information of the shape and the deformation of the sample. In order to calculate the out-of-plane field of displacement we retrieve the reference to the deformed state as:

$$
\begin{equation*}
\Delta z(x, y)=z_{D}(x, y)-z_{R}(x, y) \tag{3.12}
\end{equation*}
$$

where the reference state can be considered as the original shape of the sample, thus the difference is directly the deformation.

### 3.6.2.2 In-plane displacement

For the calculation of the in-plane displacement the same images of the 4 -step algorithm where used. To do so, the fringes were retrieved by calculating the mean of the set of 4 images for both, the reference and the deformed set.

$$
\begin{align*}
& I_{R}(x, y)=\frac{\sum_{i=1}^{N} I_{R i}(x, y)}{N}  \tag{3.12}\\
& I_{D}(x, y)=\frac{\sum_{i=1}^{N} I_{D i}(x, y)}{N} \tag{3.12}
\end{align*}
$$

Where $N$ is the number of steps utilized in the algorithm (in our case 4). Both images where processed utilizing the template matching methodology described in 3.2.2.

### 3.6.3 Experimental results

Figures 3.6 and 3.7 show the latex template sample and the deformation generated by the stress applied along the optical axis of the system (z-axis). The load was applied using a spherical object with a controlled displacement of 12.5 mm .


Figure 3.7 Speckle pattern printed on the sample: a) undeformed, b) deformed.

In the case of the displacement measurements for $x$ and $y$, the DIC technique was utilized, with these parameters: Template size $=25 \times 25$ pixels, analysis window size $=50 \times 50$ pixels, xpitch $=$ 11.79 pixels and ypitch $=9.23$ pixels.

A second order polynomial shape function was utilized to obtain subpixel resolution of the motion vectors.

The corresponding fields of displacement were calculated using both techniques. Figure 3.8 present the results for each of the axial deformations obtained by the displacement. Since the displacement of a sphere is a well-known deformation, it can be seen that the technique has a high accuracy. Each point in the graphs represents the position of that point of interest in the reference image. Then, the color associated to that point represents the displacement suffered by that pixel in the direction of the displacement map where its ploted.


Figure 3.8 Fields of deformation: a) $x$-axis, b) $y$-axis, c) $z$-axis.

### 3.6.3 Conclusions

The techniques of DIC and FP were utilized to calculate the deformation fields of a latex sample. Since DIC has high sensitivity to the in-plane deformations and FP presents great resolution in out-of-plane shape and deformation measurements, the benefit of combination of both techniques is
obvious. With this methodology it is possible to calculate the 3D motion vector for each point on the sample without the use of a Stereo-DIC system. With the capability of the system is possible to implement a non-invasive finite element analysis in applications of engineering.

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## 4. Stereo vision

### 4.1 Introduction

Stereo vision (SV) is one of the most utilized methods for the analysis of 3D scenes. The principle of the technique is based in the nature of human vision: the sensation of depth is obtained through the correlation of two perspectives of the same scene (Stereopsis) [1]. This chapter presents a general review of the most utilized arrangements and the setup proposals. Section 4.2 describes a system to locate highly reflective reference points using an emulated one-camera stereo vision system and a twin camera system. Section 4.3 explains the principles of stereo vision using a biprism and the considerations to correct the error induced by the biprism aberrations.

### 4.2 Spatial location of reference points over an object using stereo vision

One of the applications of SV is the $360^{\circ}$ scan of an object in order to obtain topography details or to get a spatial position of a given set of points of interest. However, the required computational time is too slow to perform real-time measurements in a full-field measurement. In this section we present an alternative approach based on Hering's coordinate system and the use of high reflective markers, used as reference points to track an object movement. The advantage of using these markers is that their detection is faster than a full scene correlation since it is performed by matching the position of the centroids of each marker without using an image correlation analysis.

### 4.2.1 Parallel axes geometry

The stereo vision technique is very useful in getting information about the depth of points of the same object. The capability of perceiving the depth is called stereopsis and it is obtained by capturing two images with two cameras separated by a certain distance called Stereo-system baseline. The optical system is shown in figure 4.1.


Figure 4.1 Experimental arrangement for stereo vision technique.

The experimental setup consists of two cameras which capture an image of the same object but from different perspectives. The cameras are separated by a distance $b$ (baseline). Both cameras are located at a distance D from a virtual reference plane which is tangent to the highest point of the object. The following equation is used to calculate the depth of any point of interest of the object with respect to the virtual reference plane [2]:

$$
\begin{equation*}
z=D \frac{f}{d_{1}-d_{2}} \tag{4.1}
\end{equation*}
$$

where: $z$ is the depth of the point, $D$ is the distance between the two camera planes and the virtual reference plane, $f$ is the focal distance of the camera, $d_{1}$ and $d_{2}$ are the distances from the optical axis of the camera and the image to the point of interest on the sensor plane of each camera.

Each point of interest in the image of the two camera sensors has a given position (namely $\mathrm{d}_{1}$ and $d_{2}$ ) with respect to the camera optical axis. The two distances $d_{1}$ and $d_{2}$ directly depend on the spatial location of the points. The difference between these distances is called disparity. To obtain the correspondence of the points, a correlation process is needed (exposed in Chapter 3). With the position of the correspondent points, their minimum Euclidian distance is calculated, giving as a result the disparity.

The depth of the object points of interest is obtained from the disparity of the two image points in the two images captured by the cameras. By using this approach it is possible to obtain a point cloud which defines the shape of the object.

### 4.2.2 Hering Coordinates

The Hering coordinates are an alternative expression of the parameters involve in a stereo vision system. Instead of using Cartesian coordinates, Hering coordinates simplify the expression of the location of an object with respect to a convergent binocular system [1].

In Figure 4.2a the variables describing a binocular head vision system are shown. The presented stereo vision corresponds to a convergent axes arrangement. In this case we assume that the cameras are epipolar to the ( $x, z$ ) plane. The viewing direction of the cameras, $\varphi_{l}$ and $\varphi_{r}$, are defined as the angle between the optical axis and the axis defined by the camera position and fixation point respectively. Positive angles are measured clockwise. Instead of using these viewing directions ( $\varphi_{l}$ and $\varphi_{r}$ ) we will use the quantities such as [1]:

$$
\begin{gather*}
\alpha=\varphi_{l}-\varphi_{r}  \tag{4.2}\\
\gamma=\frac{1}{2}\left(\varphi_{l}+\varphi_{r}\right) \tag{4.3}
\end{gather*}
$$

These quantities are known as the Hering-vergence ( $\alpha$ ) and Hering-version $(\gamma)$. Vergence value is 0 when cameras axes are parallel. For each vergence, $\alpha<0$, a circle can be drawn through the nodal points and the intersection point of the two camera axes. This circle is called the Vieth-Müller circle (VMC) of vergence $\alpha$, this circle is shown in Figure 4.2a, and has the radius of:

$$
\begin{equation*}
R=\frac{b}{2 \sin \alpha} \tag{4.4}
\end{equation*}
$$

As it can be seen in figure 4.2, version is defined as the average (or cyclopean) viewing direction of the two cameras, in a more formal way, when both cameras have a fixation point $f$ with a vergence $\alpha$, the half-way spot of the baseline between the two nodal points (cameras) is called cyclopean point and the visual direction from this point to the fixation point is the Hering-version.


Figure 4.2 Convergent axes arrangement. $f$ : fixation point; $l$ and $r$ : view points; $c$ : cyclopean point; $p$ : point of interest; $\alpha$ : vergence; $\gamma:$ version, $b$ : baseline distance. $\alpha_{f}$ and $\alpha_{p}$ are the vergence of the fixation point and the point of interest. $\beta_{l}$ and $\beta_{r}$ are the angles between the fixation point and the point of interest.

On the other hand, by trigonometry the Hering $\alpha, \gamma$ coordinates can be turn into Cartesian $x, y, z$ coordinates by using this transformation:

$$
H(\alpha, \gamma)=\left[\begin{array}{l}
x  \tag{4.5}\\
y \\
z
\end{array}\right]=\frac{b}{2 \sin \alpha}\left[\begin{array}{c}
\sin 2 \gamma \\
0 \\
\cos \alpha+\cos 2 \gamma
\end{array}\right]=R\left[\begin{array}{c}
\cos \varphi_{r} \sin \varphi_{l}-\cos \varphi_{l} \sin \varphi_{r} \\
0 \\
2 \cos \varphi_{r} \cos \varphi_{l}
\end{array}\right]
$$

The assumption of $y=0$ implies that the cameras are located at the same height between them, so it can be said that there is no vertical disparity between the two perspectives captured by the cameras. In practice the problem is to locate the vergence and version of each point of interest with respect to the fixation point. The viewing angles $\beta_{l}$ and $\beta_{r}$ are utilized to calculate the disparity $\delta$ and the eccentricity $\eta$ of each point:

$$
\begin{gather*}
\delta=\beta_{l}-\beta_{r}  \tag{4.6}\\
\eta=\frac{1}{2}\left(\beta_{l}+\beta_{r}\right) \tag{4.7}
\end{gather*}
$$

A positive disparity $\delta$ denotes that the interest point is located in front of the fixation point, and a negative one means that it is located behind the fixation point. While the eccentricity determinates if the interest point is located to the right or left of the reference point.

### 4.2.3 Experimental results

### 4.2.3.1 Emulated system with one camera

In this experiment the stereo system was emulated as shown in figure 4.3 by using only single 8 bit monochromatic FireWire CCD camera with $659 \times 494$ pixels array, a lens with 8 mm of focal length and rotating plate with an accuracy of 1 minute per step. The sample was rotated in order to have two views of the object to locate the markers.


Figure 4.3 Optical arrangement with a rotating mount used to emulate stereo vision.
Images of the sample were captured sequentially by rotating the mount at intervals of 8 degrees. Once the different views of the test object were taken, we consider pairs of images to have the two necessary perspectives to perform stereo correlation. The angles of vergence and version were measured from the angle of the mount rotation used in the optical arrangement showed in figure 4.4. The sample was scanned at $360^{\circ}$. The accuracy of the method depends of the angular displacement between the views of the object.

To avoid the uncertainty of the measurement due to the depth of focus, the diaphragm of the camera was closed and we used a high frequency lamp to illuminate the sample and to improve the brightness of the high reflective markers. Figure 4.4 shows: a) the scanned object with the markers, b) the image captured by the camera with the bright markers and c) 3D plot of a few detected points. In figure 4.4a, the spots with no labels belong to points of hidden perspectives.


Figure 4.4 a) Sample with markers, b) Image captured by the camera, c) Detected points around the sample shown in a 3D map.

This methodology is very practical for a scan of $360^{\circ}$ and it is easy to make the mapping of the centroids because we know the angle of rotation of the object. However this method cannot be implemented in real time due to the need of sample rotation to simulate stereo vision.

### 4.2.3.2 Optical system with two cameras

Figure 4.5 shows the second implemented system. In this case, two cameras (with the same specifications mentioned in section 4.1.4.1) simultaneously captured two different perspectives of the object.


Figure 4.5 Optical system for stereo vision: 1) CCD cameras, 2) High frequency lamp.


Figure 4.6 a) Hand used as test object, b) 3D plot of the spatial position of each of the markers. The coordinate $(0,0,0)$ of the system corresponds to the fixation point of the system.

The test with this system consisted in an identification and tracking of the markers, we used the hand of a volunteer which was placed in front of the optical system for locating the markers. The test object is shown in figure 4.6a. The parameters of the geometry of the arrangement such as view angles and the distance from the cameras plane to the fixation point were considered in the program to monitoring of the hand. One given position of the hand was captured by the CCD camera. The digitization of the markers placed onto the hand is shown in figure 4.6b. It can be observed the spatial distribution of the markers. This is a special case in which the points of the test object are near to the plane containing the fixation point.

To obtain the measurement error, we measured an object whose dimensions are known. The object was built by stacking some blocks, see figure 4.7. The distances between the blocks were measured with a caliper. The measurements obtained with optical system shown in figure 4.5 , are compared with the target positions measured with the caliper. The difference between the two values is considered the error in the measurement.


Figure 4.7 Stacked blocks which have been marked and numerated for measurement of depth between them.

Figure 4.8 shows the markers positions detected by the stereo system. The whole object was translated in different known positions in order to evaluate the error propagation with respect to the fixation plane. Figure 4.9 shows the measurement error for markers 1 and 3 which are the farthest from the fixation point. It is observed that the error in the spatial localization of the markers increases as much as the markers depart from fixation plane.


Figure 4.8 Markers location plots: a) $x z$-plane, b) $x y$-plane, c) 3D plot.
As shown in figure 4.9, the error is larger for the markers 1 and 3 due to the fact that they are localized farther from the fixation point plane. This test was done by moving the sample to different positions, from 650 mm to 750 mm distance from the cameras. The line in 720 mm indicates the distance where the fixation plane is located. It can be seen in Figure 4.9 that the error is reduced when the markers are located near the fixation point. The localization the markers 1 and 3 with respect to the fixation point plane is of 750 mm and 700 mm respectively. The maximum error was approximately $5.6 \%$ which is associated for marker 1 .


Figure 4.9 Graph of error in the spatial localization of markers 1 and 3.
Since we used a pin-hole camera model, the error expressed in figure 4.9 is associated with the error introduced by distortion of the lenses and with the distance of the points with respect of the fixation point. The alternative methods to reduce this error are: 1) to correct the image distortion caused by
the lens and 2) use an iterative algorithm to correct the position of the markers with the respect of their distance of the fixation point.

### 4.2.4 Conclusions

We have implemented the stereo vision technique based on the Hering coordinate system for point detection and depth measurement in a scene. A scanning was performed over $360^{\circ}$, markers were placed onto an archaeological piece, camera was stationary and object rotating. The stereo system that uses a single camera has this disadvantage that cannot be used for real-time measurements.

Qualitative results were obtained in the experiment of tracking the movement of a hand. In this case the optical system comprises two cameras. High reflectance markers were placed on the test object and their centroids were spatially detected by using the stereo system. The technique detects the trajectory of each point in real time.

In order to evaluate the error of the stereo vision technique that uses two cameras, the results were compared with reference values obtained with a caliper. A maximum error was obtained for the points localized beyond the reference plane that contains the fixation point. This system has a volume of work of approximately 10 cm around the fixation point with a maximum error of $5.6 \%$.

The detected markers will be used to create virtual reference planes in profilometry techniques where it is not possible to have a physical reference plane.

### 4.3 Stereo-Digital Image Correlation (DIC) measurements with a single camera using a biprism

### 4.3.1 Introduction

Several designs of compact pseudo-stereo systems using one single camera have been proposed in literature. Two virtual stereo-views of the same object can be captured by a single camera by using two differently angled planar mirrors, a glass plate at two different rotational positions, two fixed mirrors and a third rotating mirror, ellipsoidal, hyperboloidal and paraboloidal mirrors, etc. (see e.g. [3] and references therein for an extensive review of some representative single-camera stereo systems along with their performances and limitations).

In this work we present a single-camera stereo system that uses a biprism in front of the objective to split the image in two sub-images corresponding to two virtual lateral stereo-views in the two halves of the sensor. Similar basic designs for binocular and trinocular stereovision systems using prisms have been already proposed in the computer vision literature [4,5]. However, these systems present two main limitations: (i) image formation scheme is formulated on the basis of simplifying assumptions on the relative position and orientation between camera and prism, and (ii) the significant amount of image distortion introduced by the prism is ignored.

Our work aimed to overcome these two limitations by developing and testing a biprism-single camera (BSC) system suitable for highly accurate time-resolved deformation measurement via DIC. In particular, we adopted a robust generalized stereo-system calibration framework that is insensitive to camera/biprism misalignments and we implemented a model-free optimization based
procedure to map and correct the image distortion error introduced by the biprism over the entire measurement volume.

A large variety of model-based approaches have been proposed in literature to deal with image distortion removal when using optical systems with low-cost video optics or wide-angle and fisheye lenses [6-10]. Using a lens distortion model implies the definition of a global rule to correct the entire image. Most commonly the distortion is modeled as a parametric non-linear polynomial function of the distance of the generic point of the image to the distortion center (r). The typical formula combines the three major components of lens distortion: the radial, the decentering and the thin prism distortions [7]. The distortion parameters can be estimated along with the intrinsic and extrinsic parameters of the camera model via non-linear optimization schemes, or, alternatively, they can be calculated after the camera has been calibrated according to an idealized pin-hole model (see [11] for a more detailed survey and a quantitative comparison performances of the most frequently used calibration and model-based distortion correction techniques).

More recently, model-free approaches based on local correction of the distortion have been presented in literature in an attempt to improve the performances of existing model-based methods [12] or for dealing more accurately with complex imaging systems such as stereo optical microscopes [13] and scanning electron microscopes [14,15]. In fact, although the classical parametric model-based methods have been demonstrated to be a sophisticated approach for common digital cameras and off-the-shelf objectives, they appear to be inadequate when strong local gradients of distortion are present [13-16].

In this work, a visual inspection of the original and after distortion correction images revealed that the biprism induces a distortion pattern which does not correspond to any of the pre-defined parametric functions commonly used for correcting lens distortion. Hence, we chose to implement a model-free approach by calculating a 'piece-wise' function describing the local character of the image deformation over the entire measurement volume. To this end, a 3D set of fiducial points was generated through multiple acquisitions of a planar target sequentially positioned in order to 'scan' the entire volume of interest. In this way, along with the distortion induced by the biprism, other kinds of distortion effects depending on the surface of the sample (e.g. in the case of highmagnification view) are automatically included and corrected.

### 4.3.2 Materials and methods

### 4.3.2.1 Experimental set-up

Figure 4.10 schematically illustrates the rationale behind the proposed pseudo stereo-system. When a biprism is placed in front of a single CCD camera, two different views of the object behind the biprism are simultaneously imaged on the two halves of the sensor (figure 4.11). In an ideal perfectly aligned system, by performing a simple ray-tracing procedure, it is possible to find the exact position and orientation of the two virtual stereo-cameras as a function of the characteristics of the biprism (index of refraction $n_{\rho}$, angle $\alpha$, thickness $s$ ) and of the relative distances of objectbiprism ( $d_{o}$ ) and camera-biprism ( $d_{c}$ ) [4]. The stereo-angle of the virtual-stereo system is directly related to $\alpha$, while the lateral separation between the two sub-images on the two halves of the sensor depends on $s, d_{o}$ and $d \mathrm{c}$. In this work, the theoretical formulation of the image formation [4] was used only during the preliminary stage of the system design to find the geometry of the biprism best
suited for the camera characteristics and the dimension of the sample of interest. Indeed, one of the objectives of this work was to establish a calibration procedure that does not rely on prior knowledge of any of the above mentioned parameters to find the position and orientation of the two virtual cameras of the BSC system.


Figure 4.10 Image formation scheme for the biprism-single camera (BSC) pseudo-stereo system.
Figure 4.12 shows a picture of the final experimental set-up. A biprism $\left(n_{p}=1.4585, \alpha=161.41, \mathrm{~s}\right.$ $=30 \mathrm{~mm}$, surface quality $=40 / 60$, surface accuracy $=\lambda / 4$, manufactured by Centro de Investigaciones en Óptica, León, Gto., MX) is fixed on the top of two lab jacks stacked in order to allow vertical adjustment through a total distance of 84 mm . A $1280 \times 10248$-bit B/W CCD camera equipped with a zoom 7000 macro NAVITAR objective (focal length $18-108 \mathrm{~mm}$, F-stop 2.5-28C) is placed onto a series of mounts that allows the camera to be translated, rotated and tilted for fine adjustment with respect to the biprism. The object (in figure 4.12 a cube target with a regular square dots pattern on two adjacent faces) is placed onto a motorized rotational mount (range $=360^{\circ}$,
resolution $=0.01^{\circ}$ ) fixed to a motorized long travel translation mount (travel range $=100 \mathrm{~mm}$, resolution $=2.5 \mu \mathrm{~m}$ ). Lighting is provided by a dual gooseneck fiber optic illuminator equipped with a red filter (centered on $\lambda=630 \mathrm{~nm}$, bandwidth $\Delta \lambda=20 \mathrm{~nm}$ ) to reduce the chromatic aberration introduced by the biprism. An auxiliary twin-camera is placed at an angle of about $20^{\circ}$ with respect to the primary camera. This served to provide a standard lateral stereo-system used as reference for set-up and evaluation of the relative merits of the BSC system.


Figure 4.11 A cube target with a regular dot pattern as seen by the BSC system.


Figure 4.12 Picture of the experimental set-up.

### 4.3.2.2 BSC system alignment and calibration

The first step of the experimental procedure consisted of aligning the primary camera with the biprism. First, the two cameras were set on identical parameters, positioned at the same height and pointed at the object from two angled views. Then, the biprism was positioned in order to be fairly centered to the lens objective of the primary camera. Finally, by operating the multiaxial mounts, both the primary camera and the biprism were finely adjusted such that the front edge of the biprism split the camera field of view in two halves and the back face of the biprism and camera sensor were reasonably parallel (according to the layout in figure 4.10). Since the system calibration procedure proposed here is not based on the knowledge on the mutual position and orientation of camera and biprism, this preliminary alignment is not strictly required although it is recommended for obtaining evenly symmetric sub-images thus improving the overall accuracy of the measurement.

The second step of the experimental procedure consisted in calibrating the BSC system. In computer-vision-based measurement, camera calibration allows the 3D information to be inferred from the 2D information coded into the images. The camera model here considered is the pin-hole model, i.e. the camera is assumed to perform a perfect perspective transformation. As a matter of fact, the images are affected by a serious distortion caused by the nonlinear angle variation and to the varying optical path of the non-collimated light rays through the biprism. Hence, a postcalibration procedure is needed to find the transformation (i.e. the un-distortion function) that maps the actual images onto a distortion-free image data set following the perspective camera model. The calibration/distortion correction procedure was performed through the steps described below.

### 4.3.2.2.1 Standard stereo-system calibration and reference points data reconstruction

The biprism was lowered by adjusting the lab jacks in order to clear the field of view of the two cameras. The corner of the cube with a regular square dots pattern on it (pitch $=2 \mathrm{~mm}$, labeled with $G$ in figure 4.12) was imaged by the standard stereo-system in a position along the $y$ axis, hereafter referred to as POS.0. This setup served to calibrate the two cameras by using the Direct Linear Transformation (DLT) method described in detail in chapter 2 [17]. Briefly, the DLT method uses a set of control points whose 3D coordinates in a arbitrary global reference system are known with high accuracy (i.e. the dot calibration pattern on the two faces of the cube) to find a set of 11 coefficients that are functions of the intrinsic and extrinsic parameters of each camera. In particular, these coefficients are the unknowns of the overdetermined system built on mapping each 3D control point world coordinates ( $x, y, z$ ) into its corresponding 2D image coordinates ( $\eta, \xi$ ) through a perfect perspective transformation as follows:

$$
\begin{align*}
& \left(\eta-\eta_{0}\right)=l \cdot s_{\eta} \frac{\left(M_{x x} x+M_{x y} y+M_{x z} z+\delta_{z}\right)}{\left(M_{z x} x+M_{z y} y+M_{z z} z+\delta_{z}\right)}  \tag{4.8}\\
& \left(\xi-\xi_{0}\right)=l \cdot s_{\eta} \frac{\left(M_{x x} x+M_{x y} y+M_{x z} z+\delta_{z}\right)}{\left(M_{z x} x+M_{z y} y+M_{z z} z+\delta_{z}\right)} \tag{4.9}
\end{align*}
$$

where $l$ (mm) (focal length), $\eta_{0}, \xi_{0}$ (pixel) (coordinates of the center $C$ of the sensor) and $s_{\eta}, s_{\xi}$ (pixel/mm) (scale factors) are the intrinsic parameters of the camera, while $M_{r s}(r, s=x, y, z)$ (components of the rotation matrix) and $\delta_{r}(\mathrm{~mm})$ (components of the translation vector) are the extrinsic parameters i.e. they define the position and orientation of the camera with respect to the
global reference system. By introducing the DLT parameters $L_{i}(i=1, \ldots, 11)$ that are functions of the unknown intrinsic and extrinsic parameters of each camera system, equations (4.8) and (4.9) can be rearranged as:

$$
\begin{align*}
& \eta=\frac{\left(L_{1} x+L_{2} y+L_{3} z+L_{4}\right)}{\left(L_{9} x+L_{10} y+L_{11} z+1\right)}  \tag{4.10}\\
& \xi=\frac{\left(L_{5} x+L_{6} y+L_{7} z+L_{8}\right)}{\left(L_{9} x+L_{10} y+L_{11} z+1\right)} \tag{4.11}
\end{align*}
$$

Each control point provides two equations, thus a minimum of $n=6$ control points are needed to extract the entire set of DLT parameters and hence to calibrate each camera. Usually, $n$ is chosen to be much larger than six in order to get an overdetermined system and thus to reduce the effect of experimental errors through a linear least-squares based approach.

Once the two cameras were calibrated, the standard stereo system was used to retrieve the 3D position of a new set of more than 3000 points evenly distributed over a volume of $18 \times 46 \times 34$ $m m^{3}$ (width $\times$ height $\times$ depth, along the $x, z$ and $y$ direction of figure 4.12, respectively). This 3D control points grid was obtained by first rotating the cube target about the $z$ axis to orient one face parallel to the primary camera sensor. Then, the target was translated through 17 evenly spaced increments of 2 mm around POS. 0 (POS.-8, POS. $-7, \ldots$, POS. +7 and POS. +8 hereafter); for each position, the 3D coordinates of a set of $23 \times 9$ control points on the target face was reconstructed via the previously calibrated stereo-system. A LabVIEW code allowed for automatic rotation and translation of the target by means of the two high-precision motorized mounts (labeled with N and M , respectively, figure 4.12 , maximum error of repeatability in repositioning $=\lambda / 10$ evaluated via interferometry) and for automatic capture of images at each configuration of interest.

Although the retrieved position is not the 'error-free' position of the volumetric point cloud (due to the reconstruction error $d$ of the standard Stereo-DIC measurement $d=0.021 \mathrm{~mm} \pm 0.032 \mathrm{~mm}$ calculated as the Eucledian distance between reconstructed and theoretical point positions), it served to build a volume of $m=207 \times 17$ evenly spaced control points ( 207 dots over the target face- $x z$ plane-and 17 positions of the target along the depth-y axis-) to be used as a reference in the subsequent phase of retrieval of the volumetric distortion function.

### 4.3.2.2.2 Evaluation of the volumetric distortion function

The biprism was raised back by adjusting the lab jacks in order to fill the field of view of the primary camera. A series of configurations identical to step-1 were considered but, this time, for each target position, a single image was captured from the primary camera through the biprism. Each image contained two sub-images (see figures 4.10 and 4.11) that were virtually equivalent to the two views of a lateral stereo-system. Analogously to step-1, the collected images were processed first by calibrating the virtual stereo system (with the cube target in POS.0), and then reconstructing the control grid of one single face sequentially moved through POS.-8 to POS.+8. As expected, the images captured with the BSC system present an evident distortion (visible with naked eye, see figure 4.11). Hence, when these images are processed by using an idealized pin-hole projection scheme, the 3D reconstruction is affected by a significant error. As an example, figure
4.13a shows the reconstruction error (calculated as the Euclidean distance $d$ between reconstructed points and corresponding theoretical points) for the calibration target in POS.0.


Figure 4.13 Plot of the error $d$ between theoretical 3D position of calibration points in figure 4.11 and its experimental counterpart measured with the BSC system before (a) and after distortion correction (b).

In this work we implemented a post-calibration procedure that creates a map of the 3D distribution of the reconstruction error and then corrects it on the basis of a local approach. In particular, the proposed methodology results in a data error reduction by evaluating the displacement induced by the distortion for each control marker in the image through an optimization-based approach (see scheme in figure 4.14). The optimization problem is formulated into the Matlab environment, by using the function fmincon, as follows:

- the reference position of the $j k$-th point of the $m$ volumetric cloud in the global coordinate system (retrieved with the procedure described in section 4.2.2.2.1) is $P_{j k}^{T}=\left(x_{j k}^{T}, y_{j k}^{T}, z_{j k}^{T}\right)$ (mm, mm, mm) (with $j=1, \ldots, 207$ and $k=1, \ldots, 17$ );
- the erroneous position of the $m$ control points calculated from the uncorrected BSC system images on the basis of a perfect perspective transformation is $P_{j k}^{D I S T}=\left(x_{j k}^{D I S T}, y_{j k}^{D I S T}, z_{j k}^{D I S T}\right)$ ( $\mathrm{mm}, \mathrm{mm}, \mathrm{mm}$ ) (see figure 4.14.4);
- the image of the $j$-th control marker on the calibration target face to the POS.k has coordinates $I_{R_{-} j k}^{D I S T}=\left(\eta_{R_{-} j k}^{D I S T}, \xi_{R_{-} j k}^{D I S T}\right)$ (pixel, pixel) in the $k$-th virtual right image of the BSC system. Analogously, the position of the image of the $j$-th control marker in the $k$-th virtual left image is $I_{L_{-} j k}^{D I S T}=\left(\eta_{L_{-} j k}^{D I S T}, \xi_{L_{-} j k}^{D I S T}\right)$ (pixel, pixel);
- the unknown positions of the $j$-th control marker in the distortion-free $k$-th right and left images are $I_{R_{-} j k}^{C O R R}=\left(\eta_{R_{-} j k}^{C O R R}, \xi_{R_{-} j k}^{C O R R}\right)$ and $I_{L_{-j k}}^{C O R R}=\left(\eta_{L_{-} j k}^{C O R R}, \xi_{L_{-} j k}^{C O R R}\right)$, respectively;
- the distortion due to the biprism introduces a displacement $u_{R / L_{-} j k}$ (pixel) and $v_{R / L_{-} j k}$ (pixel) between $I_{R / L_{-} j k}^{D I S T}$ and $I_{R / L_{-} j k}^{C O R R}$ along the $\eta$ and $\xi$ directions respectively (see figure 4.14.5);
- at each iteration, given $u_{R / L_{-} j k}$ and $v_{R / L_{-} j k}$ as variables of the optimization, the corrected position of the $j k$-th control marker in the virtual right image is hence defined as:

$$
\begin{align*}
& \eta_{R_{-} j k}^{C O R R}=\eta_{R_{-} j k}^{D I S T}+u_{R_{-} j k}(\text { pixel })  \tag{4.12}\\
& \xi_{R_{-} j k}^{C O R R}=\xi_{R_{-} j k}^{D I S T}+v_{R_{-} j k}(\text { pixel }) \tag{4.13}
\end{align*}
$$

and similarly for the virtual left image.

- the objective function to be minimized is the Euclidean distance $d$ between $P_{j k}^{T}$ and $P_{j k}^{C O R R}$ where the 3D coordinates of $P_{j k}^{C O R R}$ are calculated at each iteration from $I_{R_{-} j k}^{C O R R}=$ $\left(\eta_{R_{j} j k}^{C O R R}, \xi_{R_{j} j k}^{C O R R}\right)$ and $I_{L_{-} j k}^{C O R R}=\left(\eta_{L_{j} j k}^{C O R R}, \zeta_{L_{j} j k}^{C O R R}\right)$, through a perfect perspective transformation scheme.


Figure 4.14 Scheme of the optimization-based procedure for mapping and correcting the distortion error throughout the measurement volume.

The non-linear optimization procedure yields four volumetric maps of error: $u_{R}, v_{R}, u_{L}$ and $v_{L}$. For each error component (e.g., for $u_{R}(x, y, z)$, the Matlab function griddata $3\left(x^{D I S T}, y^{D I S T}, z^{D I S T}, u_{R}, x, y, z\right)$ is used to fit the $m$ non-uniformly spaced vectors $\left(x_{j k}^{D I S T}, y_{j k}^{D I S T}, z_{j k}^{D I S T}, u_{R_{-} j k}\right)$ with a hypersurface and, at the same time, to interpolate the value of $u_{R}$
at any point $(x, y, z)$ of interest over the entire measurement volume with a tessellation-based linear interpolation scheme that uses the 'local' information from the nearby control points. Figure 4.15 shows two of the four distortion maps obtained from the BSC calibration procedure (only a few sections of the volumetric data have been plotted for clarity of representation).

It is worthwhile to point out here that the corrected images of the virtual BSC stereo-system do not correspond to the images of the standard stereo-system since the two systems possess different intrinsic and extrinsic parameters.

### 4.3.2.2.3 Validation of the distortion correction procedure

The volumetric error maps in figure 4.15 can be practically considered as the un-distortion functions to be applied to a given object point $(x, y, z)$ reconstructed with the BSC system within the measurement volume. For example, if these un-distortion functions are applied to the virtual right and left images of the calibration target in the POS_0 (figure 4.11) the reconstruction error drops from $0.17 \pm 0.10 \mathrm{~mm}$ (figure 4.13 a ) to $0.09 \pm 0.05 \mathrm{~mm}$ (figure 4.13 b ), i.e. the distortion correction procedure allows 3D measurement with an accuracy comparable to that of the standard Stereo-DIC system.


Figure 4.153 D plot of the distributions of $u_{R}(\mathrm{x}, y, z)$ and $v_{R}(\mathrm{x}, y, z)$ obtained by processing the set of virtual right-hand images of the calibration pattern from POS.-8 to POS.+8. Due to the symmetry of the BSC system, the data processing for the set of virtual left-hand images yields a similar 3D distributions for $u_{L}(\mathrm{x}, y, z)$ and $v_{L}(\mathrm{x}, y, z)$.

To verify that the above derived volumetric un-distortion functions are not target-dependent and that they can be effectively applied to whatever object is arbitrarily placed within the measurement volume, a portion of a ping-pong ball with a sprayed speckle pattern on it, was imaged by the BSC system (figure 4.16a) and reconstructed after matching the two virtual views via DIC [16]. Figure $4.16 \mathrm{~b}-\mathrm{c}$ report the deviation from the reference geometry of the sample (radius $=18.265 \pm 0.065$ $m m$, measured with a spherometer in three points) for the uncorrected and corrected reconstructed shapes, respectively. Significantly, application of the un-distortion procedure reduced the error from $0.32 \pm 0.10 \mathrm{~mm}$ (figure 4.16 b ) to $5.610^{-4} \pm 0.08 \mathrm{~mm}$ (figure 4.16 c ). It is interesting to notice how
the pattern of distortion in the case of the ping-pong ball is less evident than that observed for the calibration target (compare figure 4.13a with figure 4.16b). This could be explained by considering that, conversely to the case of the calibration target where the dots are located through a centroidsearching procedure, for the DIC-based measurement, control points matching was performed by employing pixel subsets of fixed dimension (in this work we used a $21 \times 21$ pixel $^{2}$ template window and a $41 \times 41$ pixel $^{2}$ analysis window, see chapter 3 for further details on the DIC technique). This fact very likely acts as a 'smoothing' effect that mitigates the local character of the distortion without, however, affecting the effectiveness of the subsequent error correction procedure.


Figure 4.16 3D reconstruction of a portion of a spherical target with the DIC-BSC system: image of the sample (a), reconstruction error $d$ (with respect to the ideal shape) before (b) and after distortion correction (c).

### 4.3.2.3 Inflation test on a circular latex membrane

The aim of this work was to develop and test a compact pseudo-stereo system for high accuracy full-field 3D-DIC deformation measurement. Thus, the last step of the experimental procedure consisted of using the validated BSC system for tracking via DIC the 3D deformation of a latex circular membrane under inflation. The choice of the test case was based on the fact that the proposed system was conceived as a large-scale prototype of a future miniaturized handheld optical probe to be used for biomechanical applications. In-vitro inflation (bulge) experiments are in fact a common protocol adopted for testing quasi-spherical biological parts e.g. [18,19] and biological membranes e.g. [20-22].

A piece of commercial white latex from a glove (thickness $=0.16 \mathrm{~mm}$ ) was first sprayed with black paint by using a fine-tipped airbrush in order to provide the random speckle pattern needed for the DIC-based measurement. The membrane was then clamped with an O-ring to the circular window ( $f$ $=30 \mathrm{~mm}$ ) of a pressurization chamber and then progressively inflated over a pressure range from 0.05 kPa to 16 kPa . During pressurization, the single camera of the BSC system recorded a frame sequence at 11 fps to be used for subsequent deformation data processing. The BSC system later allowed for extracting the shape of the inflated membrane from each single image of the sequence as well as for tracking deformation with respect to the reference configuration (at $p=0.05 \mathrm{kPa}$ ). Due to the large deformation undergone by the membrane, a serial approach was implemented to track the deformation through the sequence of images captured with the BSC system [23,24].

Figures 4.17a-c show the distortion-free 3D full-field maps of the $u, v$ and $w$ components of the surface displacement of the inflated latex membrane (at $p \approx 13 \mathrm{kPa}$ ) along the $x_{0}, z_{0}$ and $y_{0}$ directions of a reference system centered with the undeformed circular membrane (with $x_{0}, z_{0}$ and $y_{0}$ axes lying in the membrane plane and $z_{0}$ oriented outward, see figure 4.17). In this reference system, as expected, the $u$ and $v$ maps show zero-displacement along the vertical and horizontal central axes respectively due to the geometry/load/constraint symmetry and to the fact that the test sample is an isotropic, homogenous membrane of constant thickness. Furthermore, the w map presents the characteristic polar-symmetrical pattern [20] with a maximum out-of-plane displacement of about 6 mm in the central region. Since only one image is needed to perform the 3D reconstruction, analogous deformation maps can be extracted from each image of the frame sequence, i.e. the time resolution of the measurement is equal to the frame acquisition rate of the video-system.


Figure 4.17 3D DIC-BSC deformation measurement on a circular latex membrane subjected to inflation: plot of the displacement components $u(\mathrm{a}), v(\mathrm{~b})$ and $w(\mathrm{c})$.

### 4.3.3 Discussion

In this work, we presented and discussed a biprism/single-camera arrangement as an effective alternative to a traditional two camera stereo-system. The advantage of the proposed approach is the capability to perform 3D-DIC measurements with a single camera. This implies that the matching efficiency of stereo-pairs is optimized (since the two virtual cameras have identical settings) and that camera synchronization is not an issue for time-resolved measurements. On the other hand, the area of analysis on the test sample is greatly reduced ( $44 \%$ for a circle imaged into a sensor with a 3:2 aspect ratio and $70 \%$ when using a sensor with a square aspect-ratio). Decreasing the spatial resolution of an image inevitably brings a degraded resolution of the DIC-based measurement $[25,26]$. However, this optical arrangement has been conceived to be later miniaturized to serve for a very specific application that is the development of a handheld probe for 3D inspection in dermatology. The BSC-based optical probe will perform skin examinations on circular areas through in-vivo suction-tests (this motivated the adoption of a spherical target in section 4.2.2.2-3 and of the inflation test in Section 4.2.2.3). Hence, given the smoothness of the geometry of interest and the large deformation involved in this kind of measurement, it is expected that the above mentioned loss in resolution will not significantly affect the overall quality of the measurement.

In contrast to the previous similar works in the literature [4,5], the developed measurement procedure is insensitive to camera/biprism misalignment and takes into account the significant
distortion introduced by the biprism. In particular, we proposed a completely generalized two-step approach for calibrating the system and mapping the reconstruction error over the entire given volumetric domain. Without assumption of any pre-defined distortion model, but using an optimization-based routine and a subsequent interpolation process, all points in the measurement volume are mapped into the virtual, distortion-free counter parts that follow the perfect perspective model. The distortion correction possesses a strong local character since the error interpolation is done in a 'piece-wise' fashion by using information only from nearby control points. This feature is expected to become of increasing importance in the presence of a high local distortion gradient as in the case of high-magnification measurements [13]. Moreover, the un-distortion function was demonstrated to be independent from the target used for calibration [12]. This implies that, once the system has been calibrated, if the relative position/orientation of camera and biprism remains unaltered (as in the case of the future BSC-based optical probe), the test sample can be placed arbitrarily within the measurement volume without affecting the measurement accuracy (a plastic spacer could serve to insure that the area of examination is within the calibrated volume).

### 4.3.4 Conclusions

Digital image correlation is currently the method of choice for dealing with a large variety of engineering problems since it permits acquisition of 3D-deformation information with high spatial/temporal resolution and with relatively modest investment in hardware and software requirements. In this study, we have investigated the feasibility of a compact stereo-DIC system with potential for further miniaturization and use in an optical probe for in-vivo biomechanical 3D measurements. In particular, we have designed and validated a single camera pseudo-stereosystem that uses a biprism to obtain two virtual lateral stereoviews. A model-free image distortion correction scheme demonstrated to overcome the problems related to the use of a thick prism and made it possible to perform high accuracy time-resolved 3D deformation measurements on an inflated latex membrane. The results obtained in this study are encouraging and clearly demonstrate the feasibility of the proposed approach. The capability to measure full-field 3D deformation can be particularly useful when employing inverse characterization procedures for biological membranes which may possess a considerable degree of anisotropy [20-22]. For this reason, the general feasibility of the proposed methodology for investigations in biomechanics merits further study.

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## 5. Panoramic fringe projection

### 5.1 Introduction

This technique is an adaptation of the fringe projection techniques which allows to obtain the entire topography of an object from a single perspective, avoiding the move of either the system or the object. The chapter is composed as follows: Section 5.2 presents the theory of the typical FP technique, in section 5.3 are explained methods used to obtain the phase from the distorted fringe patterns, in section 5.4 the proposed Panoramic Fringe Projection (PFP) system is presented, and both the theory and the components of the arrangement are explained in detail. Finally in section 5.5 are described the experimental setup of the system along with the obtained results.

### 5.2 Fringe projection

This is a technique where fringes are projected onto a test surface [1-4]. Figure 5.1 shows fringes with a period $d$ projected on the $x y$-plane (which will be considered as the reference plane) under an angle $\theta_{l}$ to the $z$-axis.


Figure 5.1 Fringe projection geometry. $\theta_{l}=$ projection angle, $\theta_{2}=$ viewing angle.
The fringe period along the $x$-axis then becomes

$$
\begin{equation*}
d_{x}=\frac{d}{\cos \theta_{1}} \tag{5.1}
\end{equation*}
$$

In the figure the curve $S$ represents the surface to be contoured. Let's consider the fringe that starts at point $P_{1}$, it can be seen that its projection on the surface will show a displacement to $P_{2}$. This displacement is given by

$$
\begin{equation*}
u=z\left(\tan \theta_{1}+\tan \theta_{2}\right) \tag{5.2}
\end{equation*}
$$

where $z$ is the height of $P_{2}$ above the reference plane and $\theta_{2}$ is the viewing angle. On the other hand the modulation function of the fringe pattern is equal to

$$
\begin{equation*}
\psi(x)=\frac{u}{d_{x}}=\frac{z\left(\tan \theta_{1}+\tan \theta_{2}\right)}{d / \cos \theta_{1}}=\frac{z}{d} \frac{\sin \left(\theta_{1}+\theta_{2}\right)}{\cos \theta_{2}}=\frac{z}{d} G \tag{5.3}
\end{equation*}
$$

where the introduced term $G$ corresponds to the geometry factor:

$$
\begin{equation*}
G=G\left(\theta_{1}, \theta_{2}\right)=\frac{\sin \left(\theta_{1}+\theta_{2}\right)}{\cos \theta_{2}} \tag{5.4}
\end{equation*}
$$

In practice, the fringe projection can be done by generating the fringe pattern by interfering two plane waves (i.e. collimated laser beams) or creating the pattern with a computer and projecting the pattern with a multimedia projector. The second case is the one presented here.


Figure 5.2 Fringe projection geometry using a multimedia projector and a camera.
Let's consider a camera as the imaging device of the projected fringe pattern on the target. When the camera is pointing along the $z$-axis, as shown in figure 5.2 , it can be deduced that

$$
\begin{gather*}
\tan \theta_{1}=\frac{l_{p} \sin \theta_{0}+x}{l_{p} \cos \theta_{0}}  \tag{5.5}\\
\tan \theta_{2}=\frac{-x}{l_{k}} \tag{5.6}
\end{gather*}
$$

where $\theta_{0}$ is the projection angle measured from the $z$-axis and $l_{p}$ and $l_{k}$ are the projection and camera distances respectively. This gives for the displacement $u$ and the phase $\Psi$ :

$$
\begin{gather*}
u(x)=z\left(\tan \theta_{1}+\tan \theta_{2}\right)=\frac{z}{\cos \theta_{0}}\left[\sin \theta_{0}+\frac{\left(l_{k}-l_{k} \cos \theta_{0}\right) x}{l_{p} l_{k}}\right]  \tag{5.7}\\
\psi(x)=\frac{u(x)}{d_{x}}=\frac{z}{d_{x 0} \cos \theta_{0}}\left[\sin \theta_{0}+\frac{\left(l_{k}-l_{k} \cos \theta_{0}\right) x}{l_{p} l_{k}}\right]\left[1+\frac{x \sin \theta_{0}}{l_{p}}\right]^{-2} \tag{5.8}
\end{gather*}
$$

where $d_{x 0}$ is the size of the period in $x=0$. From equation (5.7) it can be seen that the displacement $u$ becomes dependent on $x$ only through $z$, if the projector and the camera are placed at equal distance from the $x y$-plane, then

$$
\begin{equation*}
l_{k}-l_{p} \cos \theta_{0}=0 \tag{5.9}
\end{equation*}
$$

Note that the fringe period $d_{x}$ is not constant, it depends of $x$. the phase $\Psi(x)$ given in equation (5.8) becomes more prone to error as the displacement $u(x)$ exceeds $d_{x}$ with a factor much greater than 1 . This can be solved, or at least create a simple approach, by dividing equations (5.7) and (5.8) by a sum of the different values of $d_{x}$ depending of the position $x$.

The final goal of the fringe projection technique is to obtain the shape of the topography information of the object, in this case named $z$. This is deduced by inverting equation (5.8):

$$
\begin{equation*}
z(x)=d_{x 0} \cos \theta_{0}\left[\sin \theta_{0}+\frac{\left(l_{k}-l_{k} \cos \theta_{0}\right) x}{l_{p} l_{k}}\right]^{-1}\left[1+\frac{x \sin \theta_{0}}{l_{p}}\right]^{2} \psi(x) \tag{5.10}
\end{equation*}
$$

If the condition described in equation (5.9) is accomplished and the period of the fringes is considered constant as the one in $x=0$, then equation (5.10) can be simplified into

$$
\begin{equation*}
z(x)=\frac{d_{x 0} \cos \theta_{0}}{\sin \theta_{0}} \psi(x) \tag{5.11}
\end{equation*}
$$

Since the captured image from the camera contains two-dimensional ( $x, y$ ), equation (5.11) can be rewritten as:

$$
\begin{equation*}
z(x, y)=\frac{\phi(x, y)}{2 \pi} \cdot \frac{p}{\tan \theta_{0}} \tag{5.12}
\end{equation*}
$$

where $p=d_{x 0}$ is the period of the fringes which is considered constant and $\psi=\phi / 2 \pi[1]$.

### 5.3 Phase obtaining methodology

When obtaining the topography information through the fringe projection technique, by using equation (5.12), the most important parameter to be calculated is the phase $\phi$. While parameters such as the period $p$ and the angle $\theta_{0}$ can be calculated by calibration (as shown in chapter 2 ) and they only contain information of the geometry of the arrangement, which is important to create the triangulation and conversion of units, the phase term contains the information directly related to the shape of the object [2,5-7].

In a digital fringe projection system, as the one shown in figure 5.2, a computer generates the digital fringe patterns composing of stripes (which can be vertical or horizontal depending of the position of the camera and orientation of projected pattern) that are sent to a multimedia projector and then projected onto the surface of the object under study (figure 5.3a), then, the stripes are deformed due
to the object's topography (figure 5.3b). A camera captures the distorted fringe pattern and the computer analyzes the fringe images.

The image captured by the camera contains information of the intensity of the fringes on the object, a general expression of the captured pattern can be written as:

$$
\begin{equation*}
I(x, y)=a(x, y)+b(x, y) \cos \phi(x, y) \tag{5.13}
\end{equation*}
$$

where $I, a, b$ and $\phi$ are functions of the spatial coordinates. Here $a$ is the mean intensity of the pattern, $b$ defines the contrast of the fringes and $\phi$ is the phase containing the information about the deformation of the fringes and surface topography.


Figure 5.3 a) Projected fringe pattern on a flat surface. b) Deformed fringe pattern due to the object's topography.

Many different methods and algorithms were developed to measure the phase [2-7]. Next Section will focus on the ones utilized in the project.

### 5.3.1 Phase shifting

Let's consider equation (5.13). As it can be seen the only information provided by the image is the intensity $I(x, y)$, while $a, b$ and $\phi$ remain as unknowns. The phase shifting methodology is based on the calculation of $\phi$ by generating an equations system with at least of the same number of unknowns. In order to do this, equation (5.13) is modified as follows:

$$
\begin{equation*}
I(x, y, t)=a(x, y)+b(x, y) \cos \left[\phi(x, y)+\varpi_{0} t\right] \tag{5.14}
\end{equation*}
$$

where the term $\omega_{0} t$ is the modulation term of the projected fringe pattern which introduces a temporal phase shifting creating a temporal dependence of the intensity $I$.

The shift introduced by $\omega_{0}$ is proportional to the number of steps necessary to obtain $\phi$, where $\varpi_{0}=2 \pi / N$ and $N \in \mathbb{N}$. Also $t=1,2,3, \ldots, N$ creating the variations of the patterns at different shots. Figure 5.4 shows the different values of $w_{0} t$ and the effect on the pattern.


Figure 5.4 Digitally generated fringe patterns with $\omega_{0}=\pi / 2$. a) $\omega_{0} t=0$, b) $\omega_{0} t=\pi / 2$, c) $\omega_{0} t=\pi$, d) $\omega_{0} t=3 \pi / 2$.

Equation (5.14) can be also expressed as

$$
\begin{equation*}
I(x, y, t)=a+b / 2 \exp \left[-i\left(\phi+\omega_{0} t\right)\right]+b / 2 \exp \left[i\left(\phi+\omega_{o} t\right)\right] \tag{5.15}
\end{equation*}
$$

The spectrum of the signal shown in equation (5.15) appears in figure 5.5. According to Servin et al. [7], to obtain the phase information $\phi$, it is necessary to make a filter of the background illumination $a$ and the complex exponential localized at $\omega_{0}$.


Figure 5.5 Spectrum of the sequence of temporal fringe patterns.
To create the so-called filter, first we have to express the shifted intensity patterns as Dirac deltas with a $N$ number of steps. The expression of the sum of these images can be written as

$$
\begin{equation*}
I_{N}(x, y, t)=I(x, y, 0) \delta(t)+I(x, y, 1) \delta(t-1)+\ldots+I(x, y, N) \delta(t-N-1) \tag{5.16}
\end{equation*}
$$

then, a linear filter is applied which is

$$
\begin{equation*}
h_{N}(t)=c_{0} \delta(t)+c_{1} \delta(t-1)+\ldots+c_{k} \delta(t-k)+\ldots+c_{N-1} \delta(t-N-1) \tag{5.17}
\end{equation*}
$$

where the constants $c_{k}$ are complex and with them is created a frequency filter to obtain $\phi$. A general expression of this filter is

$$
\begin{equation*}
h_{N}(t)=\sum_{n=0}^{N-1} e^{i n \omega_{0}} \delta(t-n) \tag{5.18}
\end{equation*}
$$

The spectrum of this filter is expressed by

$$
\begin{equation*}
H_{N}(\omega)=F\left[h_{N}(t)\right] \tag{5.19}
\end{equation*}
$$

To have a better idea of the concept of the filter, figure 5.6 shows an example of the spectrum expressed in equation (5.19) for a 4 steps algorithm.


Figure 5.6 Spectrum of the lineal filter of 4 steps.
By applying a convolution filter with the captured images, we obtain a complex image $I_{c}$ where the only important term of the convolution is the middle one, giving as a result

$$
\begin{equation*}
I c_{N}(t)=h_{N}(t) * I_{N}(x, y, t) I(x, y, 0)+I(x, y, 1) e^{i \omega_{0}}+\ldots+I(x, y, N) e^{i(N-1) \omega_{0}} \tag{5.20}
\end{equation*}
$$

Finally the phase is calculated with the angle of the complex numbers:

$$
\begin{equation*}
\phi=\tan ^{-1}\left[\frac{\operatorname{Im}\left(I_{c}\right)}{\operatorname{Re}\left(I_{c}\right)}\right] \tag{5.21}
\end{equation*}
$$

### 5.3.2 Phase Unwrapping

After solving equation (5.13) by collecting extra information from the fringe patterns; all the algorithms (whether they are phase shifting or Fourier transform methods) result in an equation of the form

$$
\begin{equation*}
\phi(x, y)=\tan ^{-1}\left[\frac{C(x, y)}{D(x, y)}\right] \tag{5.22}
\end{equation*}
$$

where $C$ and $D$ are functions of the recorded intensity from a set of patterns at the image point $(x, y)$ where the phase is being measured. Because of the resulting values of the arctan function, the solution for $\phi$ is a modulo $2 \pi$ phase function (figure 5.7) and discontinuities occur every time $\phi$
changes by $2 \pi$. If $\phi$ is increasing, the slope of the function is positive and if it decreases, the function is negative.


Figure 5.7 Characteristic saw tooth wrapped phase function.
The term 'phase unwrapping' refers to the final step in the fringe pattern measurement process, where the step consists of phase integration along a path on the saw tooth function in order to remove the discontinuities by adding a $2 \pi$ each time the phase angle presents a jump from $2 \pi$ to zero and vice versa [1,2,8-11]. Figure 5.8 shows the data of figure 5.7 after the unwrapping process.


Figure 5.8 Continuous phase function obtained by unwrapping the data in figure 5.7
The key to have a good phase-unwrapping procedure is the ability to accurately detect the $2 \pi$ phase jumps. In the case of a noise-free wrapped phase and the data is well-sampled, it is only required a sequential scan through the data to integrate the phase by adding or subtracting $2 \pi$ at the jumps.

However in most of cases the noise present in the sampled data, is a major factor in the false identification of phase jumps. A lot of work has been focused on algorithms to create a good phase unwrapping procedure to minimize the error caused by the noise [8-10].

Another important consideration for a simple unwrapping method to work is that the data must be smooth continuous across the whole image array, and extend to the boundaries of the sample window. A phase discontinuity could be caused by a rapid change in the measurement parameter, this effect can be seen as a sudden change of the fringe spacing or a fringe discontinuity. Also a lot of work has been done in order to solve this problem $[1,2,8]$, the most common solution is the use of masks to avoid the damaged area.

As it was mentioned before, the basic principle of the phase unwrapping is to integrate the wrapped phase $\phi$ (in units of $2 \pi$ ) along the path through the data. The gradient of each pixel has to be calculated by differentiation:

$$
\begin{equation*}
\Delta \phi=\phi_{n}-\phi_{n-1} \tag{5.23}
\end{equation*}
$$

where $n$ is the pixel number. If $|\Delta \phi|$ exceeds a certain threshold such as $\pi$, then a phase fringe edge ( $2 \pi$ jump) is assumed. This jump is corrected by adding or subtracting $2 \pi$ according to the sign of $\Delta \phi$.

Although equation 5.23 establishes a process for a one-dimensional signal, such as figure 5.7, it can be extended to a two-dimensional application, which is the field needed for our application. In figure 5.9 can be seen an example of a two-dimensional wrapped phase.


Figure 5.9 Two-dimensional wrapped phase.
The methods and algorithms created to unwrap a two-dimensional phase map can be classified into two groups: path-dependent methods and path-independent methods [8-11].

### 5.3.2.1 Path-dependent methods

The simplest of the phase unwrapping methods involves a sequential scan through the data, line by line, as shown in figure 5.10, at the end of each line, the phase difference between the last pixel and the pixel in the line below is determined and the line below is scanned in reverse direction. At the end, the two-dimensional information is treated as a folded one-dimensional set [1,8,9]. One of the problems of these methods is the error propagation if the region where the scan starts contains erroneous data.


Figure 5.10 Route followed by a path-dependent method in a two-dimensional phase map.

### 5.3.2.2 Path-independent methods

A path-independent algorithm works by selecting a pixel as a central point, and then a neighborhood of $n \times n$ is selected around the pixel. Then, the section is compared to the nearest neighborhood of the same dimensions in the horizontal and vertical directions directions (up, down, left, right) as follows:

$$
\begin{equation*}
\Delta \phi(x, y)=\phi(n, m)-\phi(u, v) \tag{5.24}
\end{equation*}
$$

where $x$ and $y$ are the coordinates of the selected pixel, $n$ and $m$ are the dimensions of the window around the selected pixel, and $u$ and $v$ are the coordinates of the neighborhood to be compared with the selected pixel. If one of the differences is greater than $p$ in absolute value, a $2 p$ value will be added or subtracted to the selected central pixel [1,9-11].

### 5.3.2.3 Temporal phase unwrapping

Although it is not a classification for the methodology, the temporal unwrap is a special application of the phase unwrapping process, since the process is applied to measurements where a deformation is calculated, not the topography. In this case, the idea is that the phase of each pixel is measured as a function of time, the unwrapping is the carried out along the time for each pixel independent of the others, so it is compared with itself over and over again. The mathematical expression is a variation of equation $5.23[1,8]$

$$
\begin{equation*}
\Delta \phi(t)=\phi(t)-\phi(t-1) \tag{5.25}
\end{equation*}
$$

### 5.4 A Panoramic Fringe Projection system

The Panoramic Fringe Projection (PFP) system here presented has been developed starting from the most common fringe projection (FP) scheme in which a commercial liquid crystal display (LCD) projector is used to project a computer-generated sinusoidal fringe pattern on the surface of interest imaged by a camera. If the projector is positioned at a given angle $\theta$ with respect to the viewing direction, the fringe pattern is distorted by the shape of the object and, in the special case of collimated projection and observation from infinity, it is possible to relate the height $h(x, y)$ of a given point $P(x, y, z)$ of an object surface with respect to a reference plane (usually the $x-y$ plane,
i.e $h=z$ ) and the phase $\phi(x, y)$ through the equation (5.12), figure 5.11 shows scheme of standard FP system.


Figure 5.11 Standard fringe projection system.

(a)

(b)

Figure 5.12 Rationale of the PFP technique: a) the set-up without the axicon; b) the set-up with the axicon. Red rays indicate the projected pattern, green rays indicate the observed pattern.

To convert a standard FP measurement to a Panoramic FP system, both projection and observation need to be transposed from a Cartesian-coordinate system to a cylindrical-coordinate system. To obtain a $360^{\circ}$ view of the lateral surface of a quasi-symmetrical sample similarly to the reported in [12-14,20], a concave $45^{\circ}$ conical mirror can be positioned coaxial to a single camera by making use of a beam-splitter as sketched in figure 5.12a. In this figure, a video projector is also positioned coaxially to the camera and to the conical mirror and it is used to project a sinusoidal circular pattern that is reflected on the lateral surface of the object (here a cylinder) placed inside the cone. Although the system depicted in figure 5.12a possesses either a panoramic projection and observation, it would be not able to measure the shape of the sample since $\theta=0$, in other words, the system has no sensitivity. To introduce an angle $\theta \neq 0$ between the projection and viewing directions, a convex axicon $[15,16$ ] is here used to create a divergent conical fringe pattern with a constant pitch (i.e. with a constant sensitivity) according to the scheme sketched in figure 5.12b.

### 5.4.1 PFP data processing

Figure 5.13 reports the image formation scheme of a typical PFP measurement with the virtual camera and the virtual projector separated by an angular distance $\theta$. Here, the position of a point $P$ on the object surface (in this case a cylinder) can be expressed in the Cartesian-coordinate system $x$, $y, z$ (with $z$ axis coincident with the cone axis and the origin in the cone vertex, see figure 5.13) as $P\left(x_{P}, y_{P}, z_{P}\right),(\mathrm{mm}, \mathrm{mm}, \mathrm{mm})$ and in a cylindrical coordinate system as $P\left(\alpha_{P}, r_{P}, z_{P}\right),(r a d, \mathrm{~mm}, \mathrm{~mm})$. The point $P$ is imaged in $P_{i}\left(u_{P}, v_{P}\right),($ pixel, pixel $)$ and $P_{i}\left(\beta_{P}, \rho_{P}\right),($ rad,pixel $)$ in a 2D Cartesian and in a polar-coordinate system of the camera sensor, respectively (with the origin of the polar system located in the image centre $C\left(u_{C}, v_{C}\right),($ pixel, pixel). If the system is correctly aligned and observing from infinity (by using a telecentric lens), then $\alpha_{P}=\beta_{P}$ and $\rho_{P}=M F \cdot z_{P}$ where $M F$ (pixel/mm) is the magnification factor of the camera lens. The cylinder in the figure 5.13 is the analogous of the reference plane for a standard FP system. In fact, the projected circular fringe pattern remains unaltered when projected onto a nominally perfect cylindrical sample ( $r=$ constant) placed coaxial to the conical mirror. Analogously to standard FP, when a sample with a $r=f(\alpha, z)$ is placed in place of the cylinder, the observed fringe pattern is distorted since PFP is sensitive along the radial direction, i.e. it measures the spacing of the radius of the sample from the radius of the reference cylinder. Analogously to equation (5.12) the relation between $\Delta r$ and $\Delta \phi$ for PFP can be written as:

$$
\begin{equation*}
\Delta r(\alpha, z)=k \cdot \Delta \phi(\beta, \rho) \tag{5.26}
\end{equation*}
$$

where $k$ is a constant that is a function of the geometry of the system and that can be evaluated through calibration [3].


Figure 5.13 Scheme for PFP data deduction.
In this work, the full-field phase map of the reference cylinder and of the sample has been calculated with the Four-Steps Phase Shifting (PS) algorithm [7] with the method explained in section 5.2.1. Accordingly, the registered sinusoidal fringe pattern intensity $I_{N}$ of the $N^{\text {th }}$ frame can be written as:

$$
\begin{equation*}
I_{N}(\beta, \rho)=a(\beta, \rho)+b(\beta, \rho) \cos \left[\phi(\beta, \rho)+\omega_{0} n\right] \forall n \in[1, N] \tag{5.27}
\end{equation*}
$$

where $a(\beta, \rho)$ is the background intensity, $b(\beta, \rho)$ is the fringe amplitude, $\phi(\beta, \rho)$ is the phase and $\delta$ is the phase step. For the Four-Steps PS algorithm, $\varpi_{0}=\pi / 2$ and $N=4$. In this work, the wrapped phase distribution has been calculated by solving for $\phi(\beta, \rho)$ the system of equations (5.27) and then unwrapping it with an algorithm that uses a system of iso-phase contours for better dealing with the circular fringe pattern [11]. Finally the described coordinate transformation is made.

### 5.5 Experimental results

Figure 5.14 shows a close-view picture of the experimental set-up used in this study to obtain the optical scheme depicted in figure 5.12b. To obtain a bright collimated source of light, a slides projector was disassembled and its 240W High Performance lamp was mounted separately similarly to the scheme reported in [17]. In particular, an Infrared (IR) reflector and a heat absorber is used to reject the heat produced by the lamp, then the light is condensed and collimated and hence used to illuminate a transmissive Spatial Light Modulator (SLM) with a $800 \times 600$ pixel $^{2}$ resolution ( $32 \mu \mathrm{~m}^{2}$ pixel size) to create the sinusoidal circular fringe pattern to project onto the sample. A couple of polarizers placed before and after the SLM allows to use it in the amplitude modulation mode. Just after the second polarizer, an axicon ( 50 mm diameter and $130^{\circ}$ aperture) transforms the collimated circular pattern in a conical (divergent) pattern of concentric circles of constant pitch. The light passes through a $45^{\circ}$ beam-splitter and hits the concave surface of the conical mirror that reflects it back on the lateral surface of the sample with an angle $\theta \approx 14^{\circ}$ with respect to the viewing direction. Finally the fringe pattern on the lateral surface of the sample is imaged through the beam splitter from a 1280x1024 pixel ${ }^{2} 8$-bit B/W CCD camera equipped with a 7000 macro NAVITAR telecentric zoom objective ( $18-108 \mathrm{~mm}$ focal length).

To prove the feasibility of the proposed PFP system and the validity of the developed data processing routines, a series of tests has been performed on i) a highly accurate cylinder ( $2.49 \pm$ 0.005 mm diameter), ii) a step-wise sample (with three different radii of 2,3 and 4 mm ) and iii) a quasi-axial-symmetrical sample with two longitudinal slots. These three tests served to i) obtain the reference phase map, ii) calibrate the system (i.e. calculating the $k$ coefficient of eq.(2)), iii) obtain the phase map of a generally shaped object.

Before running the test, the system has been aligned by using a laser line (for the projection unit) and the Direct Linear Transformation method [18] for the camera. In particular, a dot-calibration pattern glued on the outward $30^{\circ}$ conical portion of the conical mirror (see figure 5.13 and $[14,18]$ for details) allowed to retrieve the intrinsic parameters (and hence the $M F$ of the lens) and the extrinsic parameters of the camera (to locate the camera with respect to the $x, y, z$ coordinate system). The relative position of the camera and the conical mirror was adjusted by acting on the gimbal mount of the mirror and the multiaxial stage where the camera was fixed until a satisfactory alignment of the axis of camera and the conical mirror was reached.


Figure 5.14 Partial close-view of the experimental set-up.
Figure 5.15 shows the image data related to the measurement on the sample pictured in figure 5.16a. In particular, from figure 5.15 a to d , it is possible to see the result of transforming the four original images (Figure 5.15a) into a single wrapped phase map (Figure 5.15b), then unwrapping it (Figure 5.15 c ) and finally obtaining the $\Delta \phi(\beta, \rho)$ distribution (Figure 5.15 d ) by subtracting the unwrapped phase map with the analogous phase distribution of the reference cylinder (not reported). The $\Delta r(\alpha, z)$ distribution was obtained by multiplying the $\Delta \phi(\beta, \rho)$ distribution for the coefficient $k=1.57 \mathrm{~mm} / \mathrm{rad}$ obtained from calibration. Since the inner data of the phase disc has less density than the outer side, a linear interpolation was applied to obtain the full radial distortion. The $z$ coordinate of the measured point was obtained by dividing $\rho$ for the magnification factor $M F$.


Figure 5.15 Image sequence of fringe pattern analysis: a) original masked image; b) phase map; c) unwrapped phase map; d) phase difference between object and reference.

Figure 5.16b-c show the results of the PFP measurement in terms of radius and error on radius with respect to the theoretical radius ( 5.8 mm external diameter, 3 mm internal diameter) for the sample in figure 5.16 b . The PFP system and data processing procedure was able to correctly measure the shape of the sample though with poor resolution due to the low sensitivity (small $\theta$ angle between projection and observation directions). Error on radius (calculated as $\operatorname{err_{r}}=a b s\left(r_{\text {nom }}-r_{\text {cal }}\right)$ between nominal and calculated radius) is $0.11 \pm 0.08 \mathrm{~mm}$.


Figure 5.16 Results of PFP topography measurement of the object in panel a): b) plot of radius and c) plot of error on radius.

### 5.6 Conclusions

In this work we presented a novel Panoramic Fringe Projection system able to reconstruct the full $360^{\circ}$ shape of an object by using a conical mirror and a conical lens to realize a panoramic observation and projection, respectively. With respect to other FP applications to whole-body measurements, the main strength point of the proposed method is the capability to capture the entire
surface of the sample with a single camera from a single viewpoint, thus avoiding time consuming merging procedures that could decrease the accuracy of the measurement. Here, a Four Steps PS algorithm has been used to retrieve the phase map of the object, however, if spatial PS algorithms are used [3], a single shot would be sufficient, i.e. time-resolved measurement could be performed. Moreover, imaging the lateral surface of a sample through a conical mirror increases the spatial resolution of one order of magnitude thus allowing to perform measurement on small-sized samples with ordinary off-the-shelf lenses $[16,19]$.

The main limitations of the technique are: 1) the fact that the sensitivity of the measurement is strictly related to the axicon geometry that cannot be arbitrarily changed due to geometrical constraints in the relative positioning of the components and 2) the sample is shape and size restrained, since they must be quasi-symmetrical and its size depends of the size of the conical mirror and collimating lens. The results of this pilot study are however encouraging since the error in shape reconstruction is relatively low (about 5\% on radius) notwithstanding a small sized sample was tested, no-high quality optics were used and any kind of correction of the non-sinusoidal projected and registered fringe patterns were applied. In view of the arguments above, we can conclude that further development of the proposed PFP method could be promising for either biomedical [19] and industrial applications, such as for reverse engineering of free form surfaces and online quality control.

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## 6. Conclusions and remarks

In this thesis three optical techniques for shape and deformation measurement ; Stereo Vision (SV), Digital Image Correlation (DIC) and Fringe Projection (FP) were reviewed. The goal of this research was to design and implement new optical setups to improve data acquisition speed or reduce the use of hardware without risking the quality of results. For SV, two proposal were presented: 1) A tracking setup which allows to locate points of interest faster because of the correlation of centroids instead of a template matching, 2) a Biprism/Single-Camera device that simulates the SV with only 1 camera, solving the twin-camera problem. In the case of DIC, The technique was mixed with FP in order to compensate its lack of resolution in out-of-plane measurements. Finally a Panoramic Fringe Projection (PFP) was proposed to capture the all-around shape of an object using FP but without moving the sample nor the system.

For precise measurements the coordinate system needs to be calibrated for each of these methods. Thus the following methods of calibration, based on the use of a calibrating pattern of known dimensions, were reviewed: the Linear Transformation (DLT) calibration and Tsai's method, . The DLT calibration can be utilized in both DIC and FP methods to calibrate the camera, but it would present a problem when calibrating a projector. Tsai's method would be perfect for a fringe projecting setup, however, it would be difficult to implement it particularly to the newly proposed panoramic fringe projection method. .

In chapter 3 the theory, the formulation and the considerations of the DIC technique were presented. The principal strength of the technique is the gradient calculation over the time (deformation), especially in-plane deformation. At the end of this chapter an experimental part was presented where the DIC and FP techniques were utilized to calculate the deformation fields of a latex sample. Since DIC has high sensitivity to the in-plane deformations while FP presents great resolution in out-of-plane shape and deformation measurements, the benefit of combination of both techniques is obvious.

Chapter 4 presents two improvement proposals in Stereo Vision. In the first part we implemented a SV arrangement based on the Hering coordinate system for point detection and depth measurement in a scene. A scanning was performed at $360^{\circ}$ around archaeological piece with markers placed on it. System was using a single camera while rotating the object. While the twin-camera problem is solved and a full scan was implemented, the stereo system has the disadvantage that cannot be used for real-time measurements.

Qualitative results were obtained in the experiment of tracking the movement of a hand. In this case the optical system comprises two cameras. High reflectance markers were placed on the test object and their centroids were spatially detected by using the stereo system. The technique detects the trajectory of each point in real time. This technique can be applied in the human motion study as a tracking system of the trajectories of different parts of the body.

To evaluate the error of the stereo vision technique that uses two cameras, the results were compared with reference values obtained with a caliper. A maximum error was obtained for the
points localized beyond the reference plane that contains the fixation point. This system has a volume of work of approximately 10 cm around the fixation point with a maximum error of $5.6 \%$.

As a result of this work, a paper [1] and a proceeding were published [2].
The second part of the chapter discusses a newly proposed biprism/single-camera (BSC) arrangement as an effective alternative to a traditional two camera stereo-system. The advantage of the proposed approach is the capability to perform a 3D measurements using DIC method with a single camera. Utilization of a single camera implies that the matching efficiency of stereo-pairs is optimized (since the two virtual cameras have identical settings) and that camera synchronization is not an issue for time-resolved measurements. On the other hand, the area of analysis on the test sample is greatly reduced ( $44 \%$ for a circle imaged into a sensor with a $3: 2$ aspect ratio and $70 \%$ when using a sensor with a square aspect-ratio). Decrease of the spatial resolution in an image inevitably brings a degradation in the DIC and even more so in DIC BSC system based on measurements [3,4]. However, this optical arrangement has been conceived to be later miniaturized to serve for a very specific application that is the development of a handheld probe for 3D inspection in dermatology [25,26]. The BSC-based optical probe will perform skin examinations on circular areas through in-vivo suction-tests (this motivated the adoption of a spherical target in section 4.3.2.2-3 and of the inflation test of latex in Section 4.3.2.3). Hence, given the smoothness of the geometry of interest and the large deformation involved in this kind of measurement, it is expected that the above mentioned loss in resolution will not significantly affect the overall quality of the measurement.

In contrast to previous similar works in the literature [5,6], the developed BSC measurement procedure is insensitive to camera/biprism misalignment and takes into account the significant distortion introduced by the biprism. In particular, we proposed a completely generalized two-step approach for calibrating the system and mapping the reconstruction error over the entire given volumetric domain. Without assumption of any pre-defined distortion model, but with optimizationbased routine and a subsequent interpolation process, all points in the measurement volume are mapped into the virtual, distortion-free counter parts that follow the perfect perspective model. The un-distortion function possesses a strong local character since the error interpolation is done in a 'piece-wise' fashion by using information only from nearby control points. This feature is expected to become of increasing importance in the presence of a high local distortion gradient as in the case of high-magnification measurements [7]. Moreover, the un-distortion function was demonstrated to be independent from the target used for calibration [8]. This implies that, once the system has been calibrated, if the relative position/orientation of camera and biprism remains unaltered (as in the case of the future BSC-based optical probe), the test sample can be placed arbitrarily within the measurement volume without affecting the measurement accuracy (a plastic spacer could be used to insure that the area of examination is within the calibrated volume).

Digital image correlation is currently the preferred method of choice when dealing with a large variety of engineering problems since it permits acquisition of 3D-deformation information with high spatial/temporal resolution and with relatively modest investment in hardware and software requirements. In this study, we have investigated the feasibility of a compact stereo-DIC system with potential for further miniaturization and utilization of it in an optical probe for in-vivo biomechanical 3D measurements. In particular, we have designed and validated a single camera
pseudo-stereosystem that uses a biprism to obtain two virtual lateral stereoviews. A model-free image distortion correction scheme demonstrated to overcome the problems related to the use of a thick prism and made it possible to perform high accuracy time-resolved 3D deformation measurements on an inflated latex membrane. The results obtained in this study are encouraging and clearly demonstrate the feasibility of the proposed approach. The capability to measure fullfield 3D deformation can be particularly useful when employing inverse characterization procedures for biological membranes which may possess a considerable degree of anisotropy [9-11]. For this reason, the general feasibility of the proposed methodology for investigations in biomechanics merits further study.

As a result of this work, a paper [12] was published.
Finally chapter 5 presents a novel Panoramic Fringe Projection system able to reconstruct the full $360^{\circ}$ shape of an object by using a conical mirror and a conical lens to realize a panoramic observation and projection, respectively. With respect to other FP applications to whole-body measurements, the main strength point of the proposed method is the capability to capture almost entire surface of the sample with a single camera from a single viewpoint, thus avoiding time consuming merging procedures that could decrease the accuracy of the measurement. Here, a Four Steps Phase Shifting (PS) algorithm has been used to retrieve the shape of the object, however, if spatial PS algorithms were used [12], a single frame would be sufficient and i.e. time-resolved measurement could be performed. Moreover, the imaging sample surface through a conical mirror increases the spatial resolution by one order of magnitude thus allowing for measurements of smallsized samples with ordinary off-the-shelf lenses [13,14].

The main limitation of the technique is the fact that the sensitivity of the measurement is strictly related to the axicon's geometry that cannot be arbitrarily changed due to geometrical constraints in the relative positioning of the components. The results of this pilot study are however encouraging since the error in shape reconstruction is relatively low (about $5 \%$ on radius) notwithstanding a small sized sample was tested, no-high quality optics were used and only some correction of the non-sinusoidal projected and registered fringe patterns was applied. In view of the arguments above, we can conclude that further development of the proposed PFP method could be promising for either biomedical [14] or industrial applications, such as for reverse engineering of free form surfaces and online quality control.

As a result of this work, a paper [15] and a proceeding were published [16].

### 6.1 Future work

After the realization of this work, new proposals emerged and they are presented as follows:

1. The implementation of the Stereo-DIC system using a biprism with a laser as a light source in order to project the speckle over the sample instead of printing it. The system would be able to resolve out-of-plane deformation, while the in-plane deformation could be only calculated if the displacement is smaller than the size of the speckle. Commercial devices such as [17] utilize a two-camera setup, so our system would be simpler.
2. Continuing with the panoramic fringe projection, a second step would be the preparation of the sample with a speckle pattern in order to, not only calculate the radial deformation, but also the displacements along the surface of the sample, fusing both techniques DIC and FP in a panoramic application as performed by [12,18-20].
3. The creation of a panoramic interferometer as proposed in [21] by collocating a conical mirror in one of the arms of a Michelson's interferometer, and then obtaining the phase shifts in a single shot by means of a polarized interferometer [22].
4. Improvement of the PFP system by using divergent illumination and retrieving the conical lens of the arrangement. This can be done by calculating the phase with the change of the period in the projected fringe pattern [23].
5. Statistic validation of the proposed techniques in this thesis with the use of Monte Carlo algorithms
6. Implementation of a DIC-based dermatoscope by miniaturizing the proposed BSC system.

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