ISSN 1054-660X, Laser Physics, 2008, Vol. 18, No. 5, pp. 632–636.

Photonic Device as a Second-Order System

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Received November 28, 2007

Abstract—The present work presents the analysis of a photonic device. Starting from a laser model, it is shown that, if a two-level simplification is used and both low pumping and no losses are assumed, a second-order control system model is obtained. This model no longer describes a laser, but only a photonic device. The system response to a step input is analyzed and the results are compared with experimental data obtained when pumping a neodymium crystal with a diode laser. Both theoretical and experimental results are then compared with the analyzed photonic device. A good qualitative agreement is shown between the analytical, experimental, and computational results.

PACS numbers: 42.55.Ah, 42.55.Px, 89.20.Kk

DOI: 10.1134/S1054660X08050137

1. INTRODUCTION

Even though classical control systems are very much studied and applied in engineering, their application to nonlinear systems such as lasers are very limited due to the fact that the Laplace transform L (the Laplace space is frequently used to solve engineering classical control systems) is a linear operator difficult to apply directly to a nonlinear set of equations like that describing a laser. Given this situation, one option is to remove the nonlinear terms present in the original equation system in order to be able to apply the Laplace transform and to obtain the typical model for a classical control system. This linearization has inevitable consequences in the interpretation of the studied system. In this paper, by applying a two-level simplification plus the loss absence supposition and a low pumping for a three-level nonlinear laser, a linear system is obtained, and, therefore, the model of a second-order control system that does not describe a laser anymore, since the pumping is way beneath the threshold level, but describes a luminescent or a photonic device. For this system, the pumping will correspond to the input signal and the emitted optical luminescence to the device output signal. The system's response to a step input is analyzed and the results are compared to the experimental results obtained using a neodymium crystal pumped with a laser diode. Both theoretical and experimental results are then compared with the results from a computational model of the analyzed photonic device. Finally, a good qualitative agreement between the analytical, experimental, and computational results is shown. In Section 2, the photonic device model is introduced; in Section 3, we discuss the experimental results; in Section 4, the computational results are shown; and in Section 5, the conclusions are drawn, discussing the qualitative agreement found in the results and its limitations.

2. MODEL

Lasers are nonlinear devices largely discussed in the literature; they generate intense research, including theoretical as well as experimental.

For a three-energy level $E_1 < E_2 < E_3$, an active medium with population densities given by n_1, n_2, and n_3, a total active center density in the medium $n' = n_1 + n_2 + n_3$, the pumping transition $1 \longrightarrow 3$ and oscillation transition $2 \longrightarrow 1$ with spontaneous transition probabilities of $1/\tau_{31}$, $1/\tau_{32}$, and $1/\tau$ between levels $3 \longrightarrow 1$, $3 \longrightarrow 2$, and $2 \longrightarrow 1$, respectively, W_p is the pumping transition probability, and W is the oscillation probability, the balance equations for each level population can be obtained [1]. Considering that the light flux density $S(t)$ is related to the photon density $M(t)$ by $M(t) = S(t)/(v\hbar\omega)$, where v is the speed of light in the medium, $\hbar$ is the Planck constant divided by 2π, and ω is the oscillation transition frequency expressed by $N(t)$, which is the population inversion density, $N = n_2 - n_1$, it is possible to carry out a so-called two-level simplification; this implies that $n_3 \ll n_1$, $\partial n_3/\partial t \approx 0$, and $\tau_{32} \ll \tau_{31}$. Thus, the following balance equations are obtained [1]:

$$\frac{dS(t)}{dt} = \sigma v S(t)N(t) - v(\eta_1 + \eta_2)S(t), \tag{1}$$

$$\frac{dN(t)}{dt} = -\frac{2\sigma}{\hbar\omega}S(t)N(t) - N(t)\left(W_p + \frac{1}{\tau}\right) + n'\left(W_p - \frac{1}{\tau}\right), \tag{2}$$

where σ is the effective section of the interaction between photons and active centers and $\sigma = B_{21}\hbar\omega/v$, B_{21} is Einstein's coefficient for the stimulated transition, and η_1 and η_2 are the medium loss and the resona-

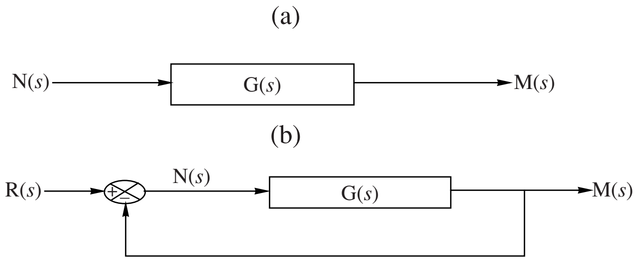

Fig. 1. (a) System with the open-loop transference function $G(s)$, input $N(s)$, and output $M(s)$ signals. (b) Feedback system with $N(s) = R(s) - M(s)$ as the input signal from the open-loop system and $R(s)$ is an external reference signal. The closed-loop transference function is then $M(s)/R(s) = G(s)/\{1 + G(s)\}$.

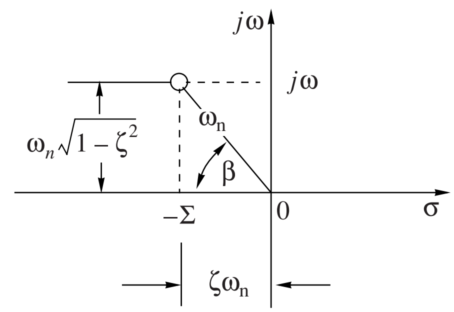

Fig. 2. Parameter representation of a second-order control system in the Laplace complex plane.

tor mirrors R_1 and R_2, respectively, of the loss reflectivity coefficients.

Assuming that the internal and reflectivity losses are negligible $(\eta_1 + \eta_2) \approx 0$ and that the pumping and spontaneous emission probabilities are similar, $W_p \approx 1/\tau$; then, the last two terms in Eqs. (1) and (2) can be ignored. Also, considering that $S(t)$ satisfies $\sigma v S(t) \approx 1$, Eqs. (1) and (2) can be written as

$$\frac{dS(t)}{dt} = N(t), \tag{3}$$

$$\tau\frac{dN(t)}{dt} = -\frac{2\tau}{\hbar\omega v}N(t) - (\tau W_p + 1)N(t). \tag{4}$$

It is important to note that these equations, due to the two prior approximations, no longer describe a laser, but only a photonic or luminescent device, where no photon loss is present and the photon flux is small. Defining $a = \tau$, $b = 2\tau/(\hbar\omega v)$, and $c = -(\tau W_p + 1)$, Eqs. (3) and (4) can be written as

$$\frac{dS(t)}{dt} = N(t), \tag{5}$$

$$a\frac{dN(t)}{dt} = -bN(t) + cN(t). \tag{6}$$

Substituting (5) and (6), we obtain

$$a\frac{d^2S(t)}{dt^2} + b\frac{dS(t)}{dt} = cN(t). \tag{7}$$

And, taking the Laplace transform for (7), we can write

$$\frac{M(s)}{N(s)} = \frac{c}{as^2 + bs}. \tag{8}$$

$N(s)$ and $M(s)$ now are the input and output signals, respectively, for a system with the open-loop transference function $G(s)$ given by the right-hand side of Eq. (8). This is shown in Fig. 1a. Figure 1b shows the same system fed by an input open-loop signal given by $N(s) = R(s) - M(s)$, where $R(s)$ is an external reference signal defined by the user. The close loop $M(s)/R(s) = G(s)/\{1 + G(s)\}$ transference function becomes

$$\frac{M(s)}{R(s)} = \frac{c}{as^2 + bs + c}. \tag{9}$$

The previous equation represents one of the transfer equations studied most often in control theory and systems analysis [2]. Defining attenuation Σ, the natural undamped frequency ω_n, and the damping factor ζ through the relations $\omega_n^2 = c/a$ and $2\zeta\omega_n = 2\Sigma = b/a$, and taking the typical constant values for a Nd:YAG solid-

Laser parameters used for simulation

	Parameters	Values
N_T	Total ion density	1.38×10^{20} cm^3
σ	Transversal section of stimulated emission	2.8×10^{-19} cm^2
n	Refraction index	1.82
τ	Fluorescence life time	230 μs
τ_R	Trip time in the cavity	0.12 ns
$h\nu$	Photon energy	1.87×10^{-17} J
ω_0	Beam width	70×10^{-4} cm
c	Speed of light	3×10^{10} cm/s
$\pi \times r^2 \times l$	Cylindrical crystal dimensions	$\pi \times 0.04 \times 1$ cm^3
l'	Cavity length	1 cm
P	Maximum pumping power	4.04 mW
V	Pumping volume	8×10^{-5} cm^3
R_2	Output mirror reflectivity	1×10^{-2}
L	Dissipative losses	1×10^{-2}
ξ	Stokes parameter	0.75987
k	Spontaneous emission factor	1×10^{-10}

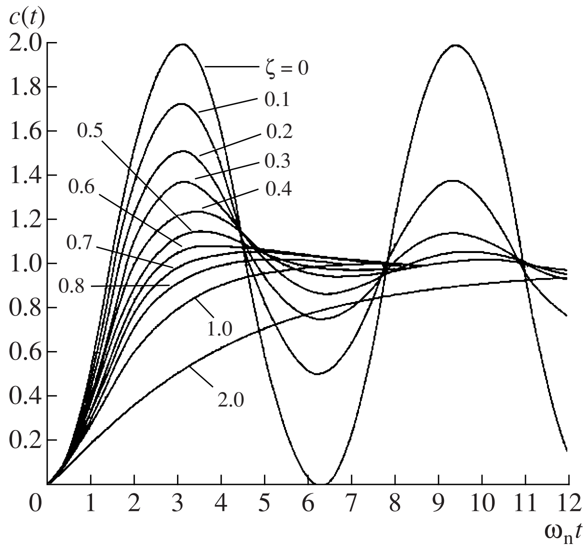

Fig. 3. Second-order control system response to a unitary step as a function of the damping factor ζ.

state laser medium [3, 4], $\sigma = 2.8 \times 10^{-19}$ cm^2, $\tau = 230$ µs, and $\lambda = 1064$ nm, the order of magnitude for the frequency, the attenuation, and the system's damping factor can be plotted in the Laplace complex plane as $\omega_n \approx \pm j \times 10^2$, $\Sigma \approx 10^8$, and $\zeta \approx 10^6$. Figure 2 shows this schematically. It is a known fact that dynamical behavior of the systems is described in terms of the parameters ζ and ω_n, if $0 < \zeta < 1$ the system is underdamped and the transitory response to a step input is oscillatory. If $\zeta = 1$, the system is critically damped and, for $\zeta > 1$, the system is overdamped. The response curves to a unitary step function is given by $T = 1/(\zeta\omega_n)$, which equals 0.6 ms for the parameters used. When the damping factor is much larger than unity, $\zeta \gg 1$, the response to the unitary step as a function of time for $t \geq 0$ can be approximated by

$$M(t) = 1 - e^{-(\zeta - \sqrt{\zeta^2 - 1})\omega_n t}. \tag{10}$$

The settling time t_s is the required time for the response to reach the range near the final value or asymptotically the step response corresponding to a ±2 or ±5% tolerance band. As a function of the time constant, $t_s = 4T$ for the 2% criterion and $t_s = 3T$ for the 5% criterion were used. To compare the experimental and computational results, we will take the $3T$ value for the settling time, that is, 1.8 ms. Due to the fact that the described system is strongly overdamped, it does not present oscillations so that the typically important parameters for underdamped systems such as the peak time and the overimpulse maximum are not applicable.

3. EXPERIMENTAL RESULTS

A cylindrical Nd:YAG crystal laser (1% concentration, 4-mn diameter, and 10-mm long) was pumped with a laser diode emitting an 808-nm centered radiation. The pumping radiation was collimated and directed to one of the Nd:Yag crystal extremes, as schematically shown in Fig. 4. On the other extreme of the Na:Yag crystal, an interference filter centered at 1.06 µm was placed to exclusively detect the radiation emitted in the medium laser transition and to block any pumping radiation at 808 nm. The results are reported in Fig. 5, where the detected signals using a digital Tektronixs model TDS 360 oscilloscope are shown, with the lower line representing the unitary pumping ramp signal at 808 nm and the upper line shows the fluorescence signal at 1.06 µm. We note that the settling time is approximately 400 µs = 0.4 ms.

4. COMPUTATIONAL SIMULATION

The computational simulation of the experimental results was achieved using a program to model the neodymium active medium laser with a system of coupled equations to describe a four-level system [5, 6]. The authors have used this model to describe neodymium lasers with a Q-modulation bar [7], as well as neodymium lasers in vanadium crystals pumped laterally with laser diodes [8]. In both cases, an excellent agreement with the experimental results was reached.

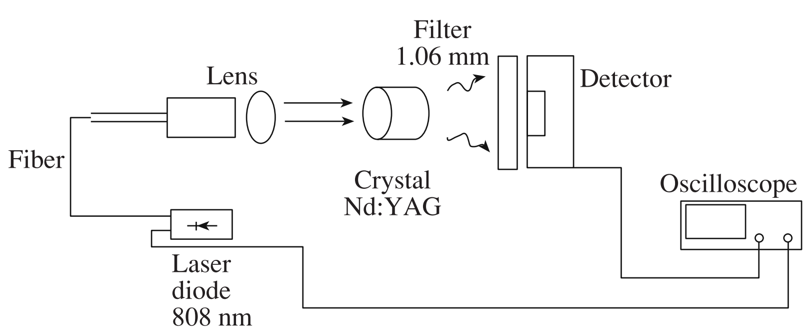

Fig. 4. Experimental setup. The laser diode pumps the active medium with an 808-nm wavelength with a step signal and the 1.06-µm emitted radiation is filtered and observed in the oscilloscope.

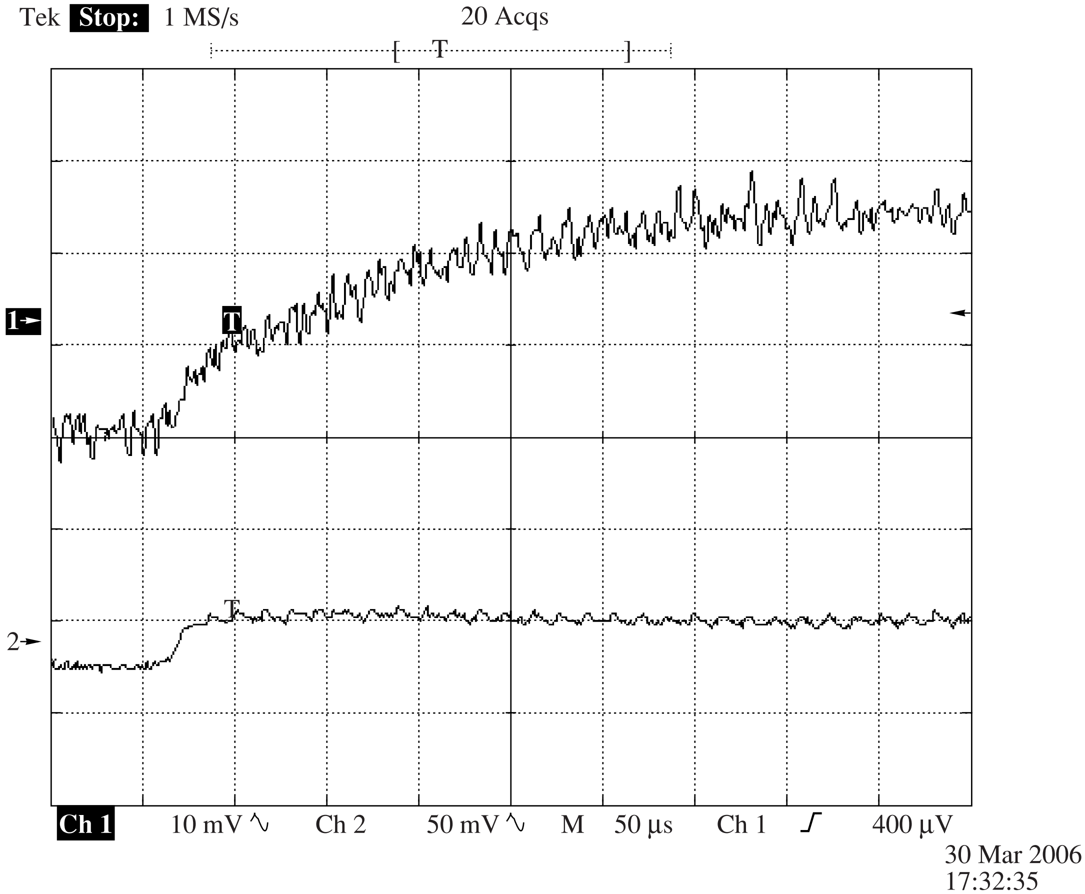

Fig. 5. Experimental results. The upper line shows the 1.06-µm emission and the lower line shows the pumping radiation at 808 nm.

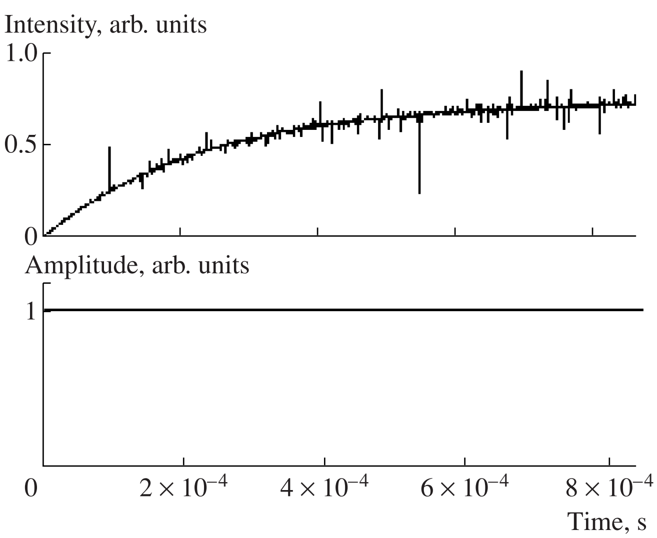

Fig. 6. Computational simulation results. The upper line shows the 1.06-µm emission and the lower line shows the pumping radiation at 808 nm.

For brevity, the program description is left out, since it is discussed in extreme detail in [7, 8]. The parameters used for this simulation are listed in the table. Figure 6 shows the computational results. The upper part of the figure shows the 1.06-µm emission and the lower part shows the proportional pumping step signal. We can see that the settling time is approximately 0.8 ms.

5. CONCLUSIONS

Beginning from a three-level laser model and a series of approximations, a photonic device is described (which is no longer a laser) as a second-order control system. It is found that for the typical values of a solid-state laser, the system is overdamped with a time constant on the order of 0.6 ms. Since no oscillations are present in the system, other important parameters for underdamped systems, such as the peak time and the overimpulse maximum, are not applicable. The analytical, experimental, and computational results for the settling time are 1.8, 0.4, and 0.8 ms, respectively, show good qualitative agreement. An important source of quantitative but not qualitative discrepancy is the fact that the model used is a three-level one, while the neodymium crystal used in the experiment and in the computational simulation is a four-level system.

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