## ANALYSIS OF INTERFEROMETRIC SENSITIVITY TO OUT-OF-PLANE OPTICAL SYSTEM

Ana Karen Reyes, Juan Antonio Rayas, Amalia Martínez-García

Centro de Investigaciones en Óptica, Loma del Bosque 115, Col. Lomas del Campestre, León, Gto. México akreyes@cio.mx

**Abstract**: In this work, the error introduced in a measurement was analyzed when, for simplicity, a constant sensitivity vector corresponding to collimated light is assumed, however experimentally a variable sensitivity vector associated with divergent light is used. Graphs were generated that indicated the error that was introduced in the measurement of the displacement when fixing variables such as: shape and size of the object and the position of the observer and the source of illumination.

**Theory**: To establish a relationship between the phase change and the displacement field, the sensitivity vector is used, which is related to the geometry of the optical system.

Fig. 1 shows the geometry for the study of the sensitivity vector. In the case of an interferometer with out-of-plane sensitivity, the measurement of the sensitivity vector  $\vec{e}$  is given by the orientation of the unit-vectors of illumination  $\hat{s}$  and observation  $\hat{b}$ :

$$\vec{e}(P) = \frac{2\pi}{\lambda} \left[ \hat{b}(P) - \hat{s}(P) \right], \qquad (1)$$





**RIAO-OPTILAS-MOPM** 

2019

where *P* is a point on the sample. On the other hand, the relationship between the phase difference  $\Delta \phi$ , the displacement vector  $\vec{d}(P)$  and the sensitivity vector  $\vec{e}(P)$  is given by the following equation:

$$\Delta \phi = \vec{d}(P) \cdot \vec{e}(P).$$
 (2)

Based on the geometry of the setup, the components of the sensitivity vector in the plane can be made negligible, so the displacement can be calculated with the following expression:

$$v = \frac{\lambda \Delta \phi}{2\pi e_z}.$$
 (3)

The relative error,  $E_{rw}$ , in the displacement of the component w is calculated as:

$$E_{rw} = \left| \frac{w_c - w_d}{w_c} \right| \times 100, \tag{4}$$

where  $w_d$  is the out-of-plane displacement by using divergent illumination and  $w_c$  is the displacement evaluation by assumption of the use of collimated. Expressing the equation. (3) to determine  $e_z$  and substitute into equation. (4), the relative error based on the sensitivity vector,  $E_r$ , is obtained by:

$$E_r = \left| \frac{e_{zd} - e_{zc}}{e_{zd}} \right| \times 100, \tag{4}$$

where  $e_{zd}$  and  $e_{zc}$  is the z component of the sensitivity vector for divergent and collimated illumination, respectively.



Fig. 3. Relative error for a square plate of different sizes: a) 25 cm<sup>2</sup> and b) 100 cm<sup>2</sup> ; ; the observer and the source of illumination are on the optical axis.



Fig. 4. Relative error for a square plate of different sizes: a) 25 cm<sup>2</sup> and b) 100 cm<sup>2</sup>; the observer and the source of illumination are on  $10^{\circ}$ .





Fig. 1. Representation of a system with out-of-plane sensitivity, where S represents the point source of illumination, B the observer position and P a point on the sample.



Fig. 5. Relative error for a convex hemisphere of radius: a) 5 cm and b) 10 cm ; the observer and the source of illumination are on the optical axis.



## Fig. 2. Topographies considered for the sensitivity vector analysis: a) Square plate and b) Convex hemisphere.

a)

b)

Fig. 6. Relative error for a convex hemisphere of radius: a) 5 cm and b) 10 cm ; the observer and the source of illumination are on  $10^{\circ}$ .

**Conclusions:** An optical system with out-of-plane sensitivity where the error in the displacement field measurement can be predicted when using divergent illumination instead of collimated illumination was designed and demonstrated using the obtained graphs. The error is dependent on the size of the object as well as the positions of the illumination source and the observer. The graphics revealed that the optical system design can be proven according to the percentage of error within the maximum acceptable range which can be established given the application.

## References

[1] H. J. Puga, R. Rodríguez-Vera, and A. Martínez, "General model to predict and correct errors in phase map interpretation and measurement for out-of-plane ESPI interferometers," Optics & Laser Technology, 34, 81-92, (2002).

[2] Thomas Kreis, Holographic Interferometry, Principles and Methods, Ed. Akademie Verlag, VCH Publishers (Germany, 1996).

[3] A. Martínez, J. A. Rayas, R. Rodríguez-Vera, and H. J. Puga, "Three-dimensional deformation measurement from the combination of in-plane and out-of-plane electronic speckle pattern interferometers," Applied Optics, 4652-4658, (2004)